VEMLab

A MATLAB library
for the virtual element method

VEMLab Primer

Version 2.3

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1 Summary of updates

From VEMLab v2.2.2 to VEMLab v2.3:

- Add possibility of using multiple materials in the Poisson module.
- Update triangulate_polygon function so that it uses the built-in triangulation matlab function.
- Bug fix in writing to txt file.
- Add possibility to write output files suitable to be read in Convex Polygon Packing (CPP) program.
- Add some options for controlling the plotting of the mesh (plot_mesh_linewidth, plot_mesh_nodes, plot_mesh_nodesize, plot_mesh_axis).
- Add sparse solver to VEM2D/FEM2D in LinearElastostatics and Poisson modules.
- Remove L-shape example from the test folder due to bad behavior in plotting stresses and strains.

From VEMLab v2.2.1 to VEMLab v2.2.2:

- Fix function max_edge_size.m.
- Add an L-shape example in the test folder.
- Update calculation of the polygon's area: MATLAB's "polyarea.m" is now used.

From VEMLab v2.2 to VEMLab v2.2.1:

- Add option to explicitly switch off all MATLAB figures in function "plot_and_output_options.m".
- Facilitate compatibility to run VEMLab in Octave.
- Update manual with a guide to running VEMLab in Octave.

From VEMLab v2.1 to VEMLab v2.2:

- Fix disp() in plot_and_ouput_options.m: disp("Hello") seems to work only in newer versions of MATLAB. So, it is changed to the standard MATLAB format disp('Hello').
- Results that are postprocessed in the graphical user interface of GiD are now ordered in folders.
- Add option to plot deformed domain in MATLAB figures when using the LinearElastostatics module (see function “plot_and_ouput_options.m” located in the folder “config”.)
- Add a function to read a meshfile having the domain type declared as "Custom", which is useful for defining the meshfile manually or using an external mesh generator or using a customized version of the mesh generators available in VEMLab. (See example "Creating and using a custom meshfile" in the VEMLab Primer for details.)
- Add more details to the VEMLab Primer.

From VEMLab v2.0.2 to VEMLab v2.1:

- Add customized wrench domain (for PolyMesher mesh generator only).
- Add customized plate with a hole domain (for PolyMesher mesh generator only).
- Add the following test: "square_plate_with_source2_poisson2d.m" in test folder.
- Add the following test: "plate_with_hole_linelast2d.m" in test folder.
- Add the following test: "wrench_linelast2d.m" in test folder.
- Fix iteration counter in PolyMesher function.

From VEMLab v2.0.1 to VEMLab v2.0.2:

- Fix several bugs when using vemlab_method='FEM2DQ4' and vemlab_method = 'FEM2DT3'.

- Add a control variable in config.m that permits to explicitly set the number of Gauss points to integrate the FEM2DQ4 stiffness matrix and body force vector.

From VEMLab v2.0 to VEMLab v2.0.1: the following features have been added

- More detailed manual in folder “doc.”
- Improvement to the plotting of axis and fonts in MATLAB figures.

From VEMLab v1.0 to VEMLab v2.0: the following features have been added

- Two-dimensional Poisson problem
- Setup of plot and output options in function “plot_and_output_options” located in folder “config.”
- Additional plotting options (stresses, strains, fluxes and gradients) to MATLAB figures, text files and GiD files.
- Option to plot solutions to VTK files.

VEMLab 1.0: (Initial release of code)

- Two-dimensional linear elastostatics (plane strain and plane stress)
- Solution methods: VEM (polygonal elements), FEM (3-node triangles, 4-node quadrilateral)
- Boundary conditions: Dirichlet, Neumann on boundary edges; can be a constant or a function.
- Meshers: PolyMesher, distmesh2d, quad4mesh; customized for rectangular domains only (requires adjustments for other domain types)
- Meshes need to be generated separately and stored inside folder “mesh_files” located in the folder “test.”
- Meshes must be generated with the functions “create_” located in the folder “mesher.”
- Solutions can be plotted to MATLAB figures, text files and GiD files.

2 Features of VEMLab

VEMLab is a free and open source MATLAB library for the virtual element method.

Features:

- Two-dimensional linear elastostatics (plane strain and plane stress) and two-dimensional Poisson problem.
- Solution methods: linear VEM (polygonal elements), FEM (3-node triangles, 4-node quadrilaterals).
- Boundary conditions: Dirichlet, Neumann on boundary edges; can be a constant or a function.
- Meshers: PolyMesher, distmesh2d, quad4mesh; PolyMesher is customized for rectangular domain, wrench domain and plate with a hole domain; distmesh2d and quad4mesh are customized for rectangular domain only. Domains can be extended for any of the meshers, but it requires adjustments to some interface functions (see the instructions that are available in functions create_polygonal_mesh.m, create_quadrilateral_mesh.m and create_triangular_mesh.m in folder “mesher”).
- Meshes need to be generated separately and saved to folder “test/mesh_files.”
- Meshes must be generated with the functions “create_” located in the folder “mesher.” Then, the files containing the generated meshes will be automatically saved to folder “test/mesh_files” for their use.
- Solutions can be plotted to MATLAB figures, text files, GiD files and VTK files.

About plotting capabilities: Currently, the MATLAB plotting of stresses and strains (linear elastostatic problem) or fluxes and gradients (Poisson problem) is very limited and won't recognize holes that might come with the domain geometries. It will work ok with rectangular geometries without holes, but the mesh will need to be very refined to obtain a colormap on the whole geometry... just try a coarse mesh and then a refined mesh to see what is being said.

If quality colormap plots are required for stresses, strains, fluxes and gradients, it is highly recommended using the GiD output files to visualize them in "GiD the pre and postprocessor" (www.gidhome.com).

On the other hand, the MATLAB plotting of primary field variables (like displacements) presents no limitations and produces quality colormaps.

3 Source code

All the information related to VEMLab and its source code is available on the web:

http://camlab.cl/software/vemlab/

Download the code before proceeding with the rest of this primer.

4 Up and running with VEMLab

VEMLab is a library. You need to create a main .m file and place it inside the folder “test.” The main file has the typical structure of a FEM simulation. Simply follow the test problems (they are given with detailed comments) that are provided inside the folder “test” to write your own .m files or modify the ones provided. More details are given below.

To run a simulation a main .m function must be prepared. This main function, when executed, will start the simulation, and drive it until its end. Some examples of main functions are provided in the folder “test.”

The folder “test/mesh_files” has various text files that contain the information of predefined meshes that are ready to be used and read from the example main files that are available in the folder “test.” To generate new meshes, the user must use the following functions that are available with instructions in the folder “mesher”:

- create_polygonal_mesh.m for polygonal elements.
- create_quadrilateral_mesh.m for four-node quadrilateral elements.
- create_triangular_mesh.m for three-node triangular elements.

The previous functions create text files that contain the information of the generated meshes and save them to the folder “test/mesh_files” so that they are available to be read from the main functions.

Important note about mesh generation: create_polygonal_mesh.m is a wrapper for PolyMesher, create_triangular_mesh.m is a wrapper for distmesh2d, and create_quadrilateral_mesh.m is a wrapper for quad4mesh; PolyMesher is customized for rectangular domain, wrench domain and plate with a hole domain; distmesh2d and quad4mesh are customized for rectangular domain only. Domains can be extended for any of the meshers, but it requires some adjustments to some interface functions (see the instructions that are available in functions create_polygonal_mesh.m, create_quadrilateral_mesh.m and create_triangular_mesh.m in folder "mesher").

The following must be considered when preparing the main file:

The method to be used in the simulation is specified by the variable vemlab_method. For instance, to perform the simulation with

- VEM, declare this variable as vemlab method='VEM2D'.
- FEM, declare this variable as vemlab_method='FEM2DQ4' for four-node quadrilateral elements, and vemlab_method='FEM2DT3' for three-node triangular elements.

The four-node quadrilateral and three-node triangular elements are particular instance of polygonal elements, and as such they can be used to simulate with the VEM specifying vemlab method='VEM2D'. However, polygonal meshes with elements of more than three edges cannot be used when specifying vemlab method='FEM2DT3', and polygonal meshes with elements of three or more than four edges cannot be used when specifying vemlab method='FEM2DQ4'.

Before starting the simulation, it is important to setup the options for the plots and output files. These options can be activated or deactivated in the function plot_and_output_options.m that is located in the folder "config."

The output files created with the simulation are saved to folder "test/output_files." Inside this folder there are three subfolders that contain specific output files, as follows: the folder "GiD" contains output files that are readable in the postprocessor of GiD (https://www.gidhome.com/), the folder "VTK" contains output files that are readable in the Visualization Toolkit VTK (https://www.vtk.org/) or in the visualization application ParaView (https://www.paraview.org/), and the folder "txt" contains the output files in text format.

5 Examples

5.1 Displacement patch test

This test consists in the solution of the linear elastostatic problem with $\boldsymbol{b} = \boldsymbol{0}$ and essential (Dirichlet) boundary conditions $\boldsymbol{g} = [x1\ x1\ +x2]^T$ imposed along the entire boundary of a unit square domain. Plane strain condition is assumed with the following material parameters: $E_Y = 1\times10^7$ psi and $\nu = 0.3$. The main file for this problem is provided as the file linear_patch_test_linelast2d.m that is in the folder "test." The polygonal mesh and the VEM results are shown in Fig. 5.1. The relative L2-norm of the error and the relative H1-seminorm of the error obtained for the mesh shown in Fig. 5.1(a) are 2.5493x10-16 and 1.3766x10-15, respectively. Therefore, as predicted by the theory, the VEM solution coincides with the exact solution given by $\boldsymbol{g}$ within machine precision.

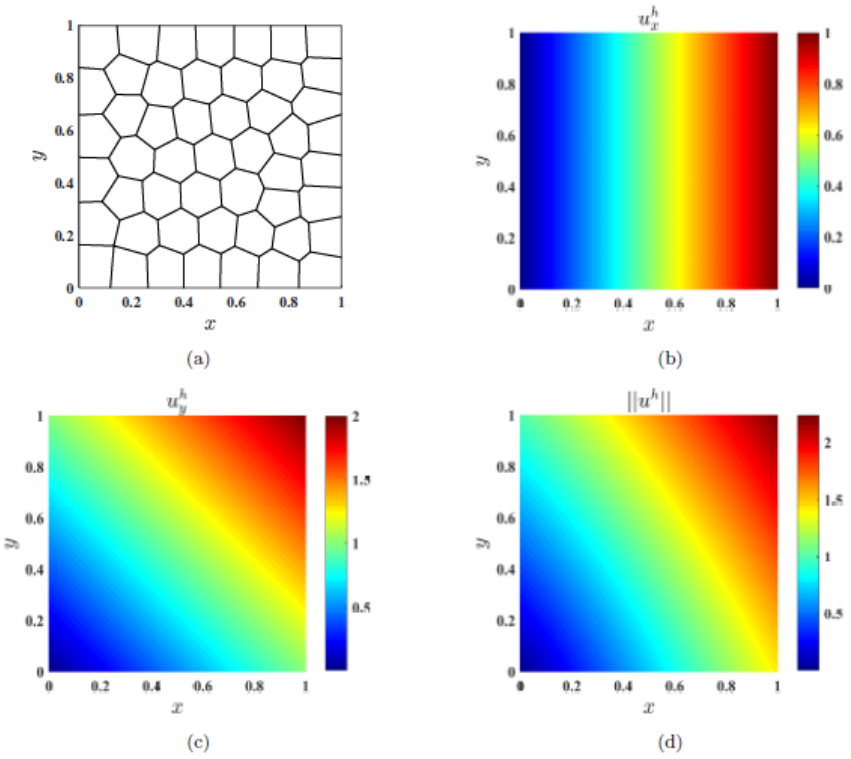

Fig. 5.1: Solution for the displacement patch test using VEMLab. (a) Polygonal mesh, (b) VEM horizontal displacement, (c) VEM vertical displacement, and (d) norm of the VEM displacement. The relative L2-norm of the error is 2.5493x10-16 and the relative H1-seminorm of the error is 1.3766x10-15.

5.2 Cantilever beam subjected to a parabolic end load

The VEM solution for the displacement field on a cantilever beam of unit thickness subjected to a parabolic end load P is computed using VEMLab. Fig. 5.2 illustrates the geometry and boundary conditions. Plane strain state is assumed. The essential boundary conditions on the clamped edge are applied according to the analytical solution given by Timoshenko and Goodier:

$$u_x = -\frac{Py}{6\overline{E}_Y I}\left((6L-3x)x + (2+\overline{\nu})y^2 - \frac{3D^2}{2}(1+\overline{\nu})\right),$$

$$u_y = \frac{P}{6\overline{E}_Y I}\left(3\overline{\nu}y^2(L-x) + (3L-x)x^2\right),$$

where $\overline{E}_Y = E_Y/(1-\nu^2)$ with the Young's modulus set to E_Y = 1×10^7 psi, and $\overline{\nu} = \nu/(1-\nu)$ with the Poisson's ratio set to ν = 0.3; L = 8 in. is the length of the beam, D = 4 in. is the height of the beam, and I is the second-area moment of the beam section. The total load on the traction boundary is P = -1000 lbf.

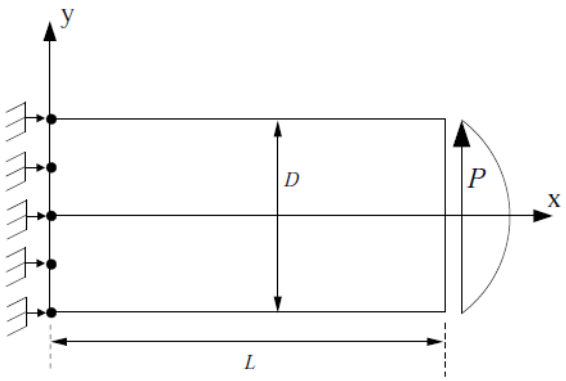

Fig. 5.2: Model geometry and boundary conditions for the cantilever beam problem.

In order to solve this problem in VEMLab, the function cantilever_beam_linelast2d.m is used. This function is located in the folder "test." The polygonal mesh and the VEM displacements results are shown in Fig. 5.3.

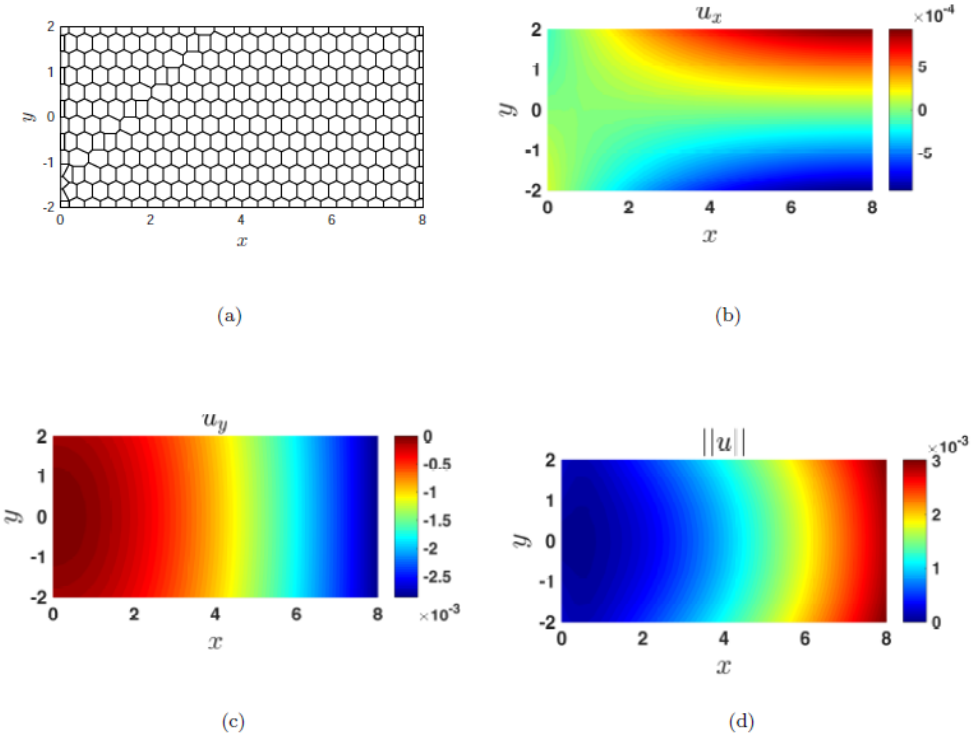

Fig. 5.3: Solution for the cantilever beam subjected to a parabolic end load using VEMLab. (a) Polygonal mesh, (b) VEM horizontal displacement, (c) VEM vertical displacement, (d) norm of the VEM displacement.

A performance comparison between VEM and FEM is conducted. For the FEM simulations, three-node triangles (T3) are used. The performance of the two methods are compared in Fig. 5.4, where the relative H1-seminorm of the error and the normalized CPU time are each plotted as a function of the number of degrees of freedom (DOF). The normalized CPU time is defined as the ratio of the CPU time of a particular model analyzed to the maximum CPU time found for any of the models analyzed. From Fig. 5.4 it is observed that for equal number of degrees of freedom both methods deliver similar accuracy and the computational costs are about the same as the mesh is refined.

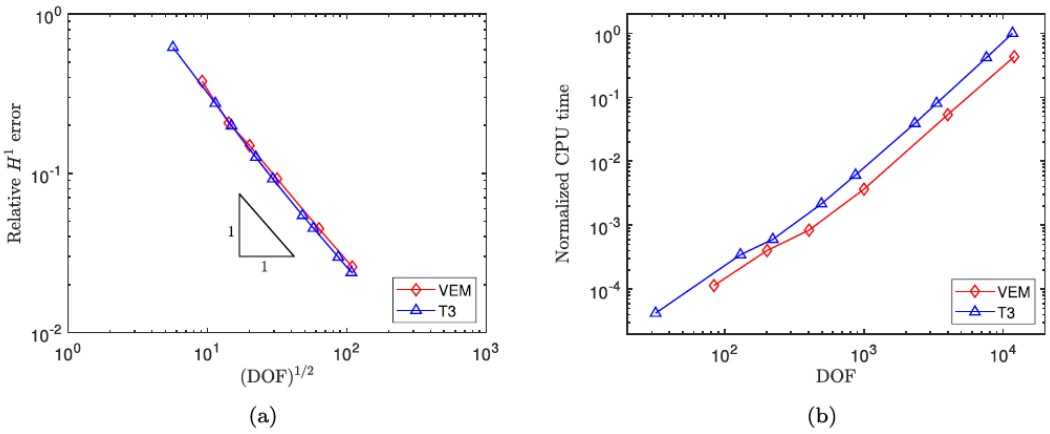

Fig. 5.4: Cantilever beam subjected to a parabolic end load. Performance comparison between the VEM and the FEM (three-node triangles (T3)). (a) Relative H1-seminorn of the error as a function of the number of degrees of freedom and (b) normalized CPU time as a function of the number of degrees of freedom.

5.3 Creating and using a custom meshfile

This example illustrates how to define a customized mesh when the mesh is not generated with the mesh generation functions provided in the source code of VEMLab.

Consider the cantilever beam subjected to an end load that is shown in Fig. 5.5. The domain is a rectangle of dimensions 27.2 x 17. The material parameters are *Ey* = 1e7 and ν = 0.3, and plane strain condition is assumed. The end load is set to -10000. The mesh has been defined manually. The clamped side is defined as the Dirichlet boundary and the loaded end is defined as the Neumann boundary. The custom mesh file starts with the keyword "Custom" and is followed, as usual, by the quantity of nodes (in this case, 44) and their coordinates; then, the number of polygonal elements (in this case 7) and their connectivity. The three final lines of the mesh file correspond, respectively, to the list of the nodes located on the Dirichlet boundary (in this case 2 nodes), the list of nodes located on the Neumman boundary (in this case 2 nodes) and the bounding box xmin, xmax, ymin, ymax, where the domain is contained. The bounding box area can be larger than the domain area. The bounding box is only used for plotting purposes using MATLAB figures --- it is not used for any other type of plot (e.g., GiD and VTK). The complete meshfile is provided in the folder "test/mesh_files" as the file "vem_letters_7poly_elems.txt" and is reproduced below. The main function used to run this problem is the function vem_letters_linelast2d.m that is in the folder "test".

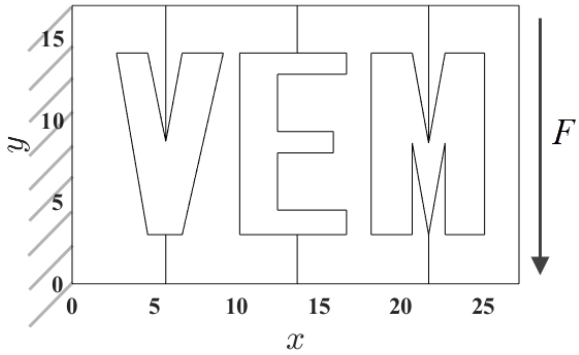

Fig. 5.5: A cantilever beam subjected to an end load and discretized with a customized mesh formed by 7 polygonal elements.

```
# domain type
Custom
```

```
# nodal coordinates: number of nodes followed by the coordinates
44
0.00   0.00
5.70   0.00
5.70   3.00
4.60   3.00
2.70   14.10
4.60   14.10
5.70   8.70
5.70   17.00
0.00   17.00
13.70  0.00
13.70  3.00
10.20  3.00
10.20  14.10
13.70  14.10
13.70  17.00
16.70  3.00
16.70  4.50
12.50  4.50
12.50  8.00
15.90  8.00
15.90  9.30
12.50  9.30
12.50  12.80
16.70  12.80
16.70  14.10
21.70  0.00
21.70  3.00
20.70  8.60
20.70  3.00
18.20  3.00
18.20  14.10
20.70  14.10
21.70  8.60
21.70  17.00
22.70  8.60
22.70  3.00
25.10  3.00
25.10  14.10
22.70  14.10
27.20  0.00
27.20  17.00
6.70   3.00
9.20   14.10
6.70   14.10
# element connectivity: number of elements followed by the connectivity (each line:
nodes_per_element(nel) node1 node 2 ... node_nel)
7
9 1 2 3 4 5 6 7 8 9
8 4 3 42 43 44 7 6 5
13 2 10 11 12 13 14 15 8 7 44 43 42 3
14 12 11 16 17 18 19 20 21 22 23 24 25 14 13
23 10 26 27 28 29 30 31 32 33 34 15 14 25 24 23 22 21 20 19 18 17 16 11
12 30 29 28 27 35 36 37 38 39 33 32 31
11 26 40 41 34 33 39 38 37 36 35 27
# indices of nodes located on the Dirichlet boundary
1 9
# indices of nodes located on the Neumann boundary
40 41
# xmin, xmax, ymin, ymax of the bounding box
0 27.2 0 17
```

6 Plotting in GiD

MATLAB plots available in VEMLab provide limited options as discussed in Section 2. GiD postprocessing is the most complete option to visualize all the variables that are computed in VEMLab. When activated in plot_and_output_options.m, results are written to GiD files and saved to folder "test/output_files/GiD". GiD can be downloaded from its webpage: https://www.gidhome.com/.

Warning: the postprocessing in GiD is made by subdividing the polygonal elements into triangles. A correct visualization in GiD requires a poygonal mesh that is formed by convex polygons. When nonconvex polygons are present in the mesh, the postprocessing of the results will not be shown correctly.

The following figures summarize the procedure to visualize the stresses in the postprocess module of GiD. The same procedure applies for any of the other variables computed by VEMLab (e.g., displacements, strains, gradient, fluxes, etc.)

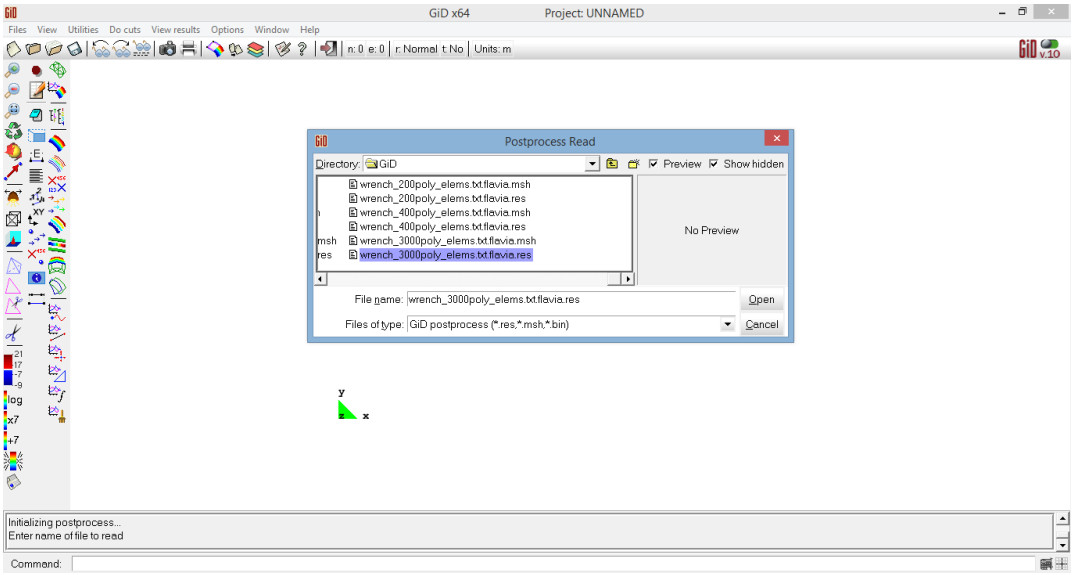

Fig. 6.1: In the postprocess module, open the .res file that was saved to folder "test/output_files/GiD."

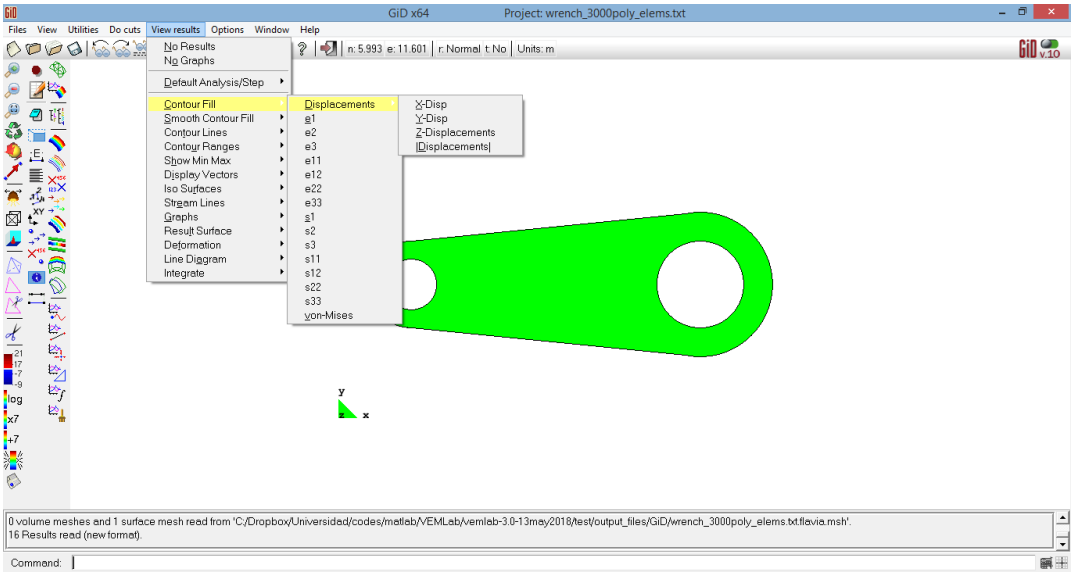

Fig. 6.2: In the postprocess module, click on "View results" to display a menu with various options for plotting results. See for instance, "Contour Fill."

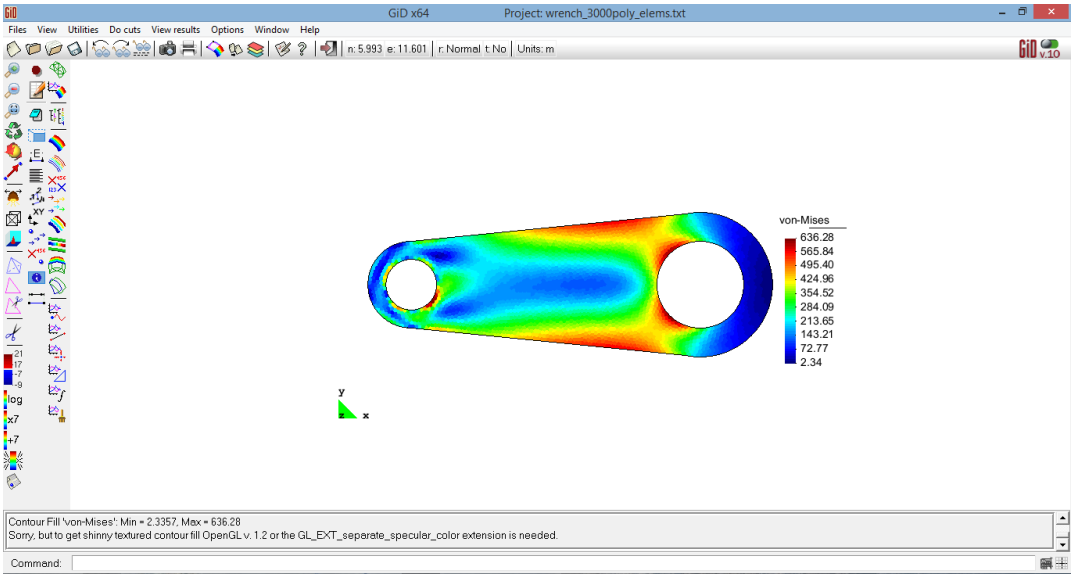

Fig. 6.3: In View results->Contour Fill, click on von-Mises to plot the von Mises stress.

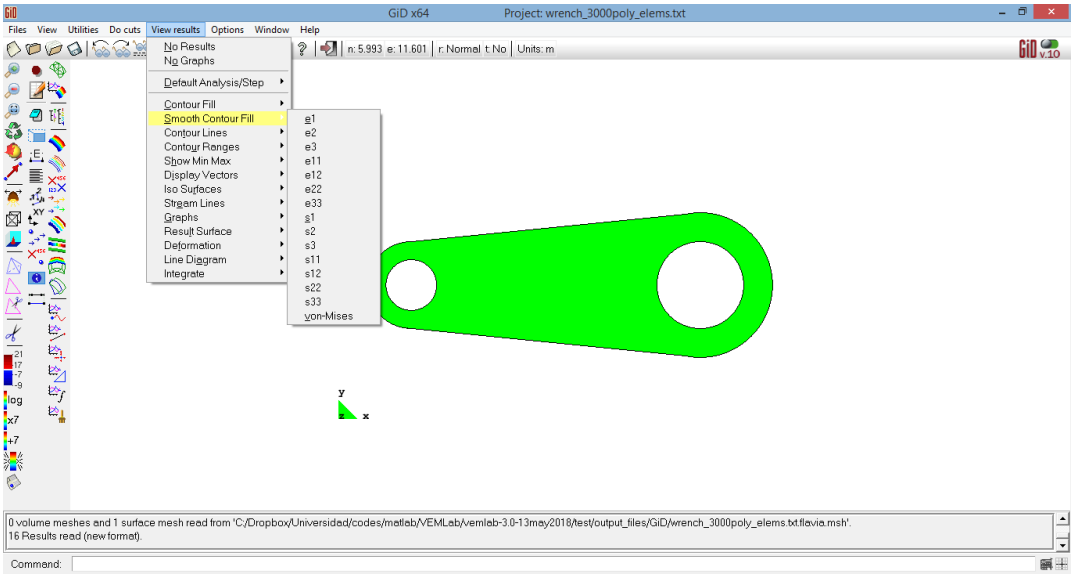

Fig. 6.4: If desired, stresses can be smoothed using the menu View results->Smooth Contour Fill.

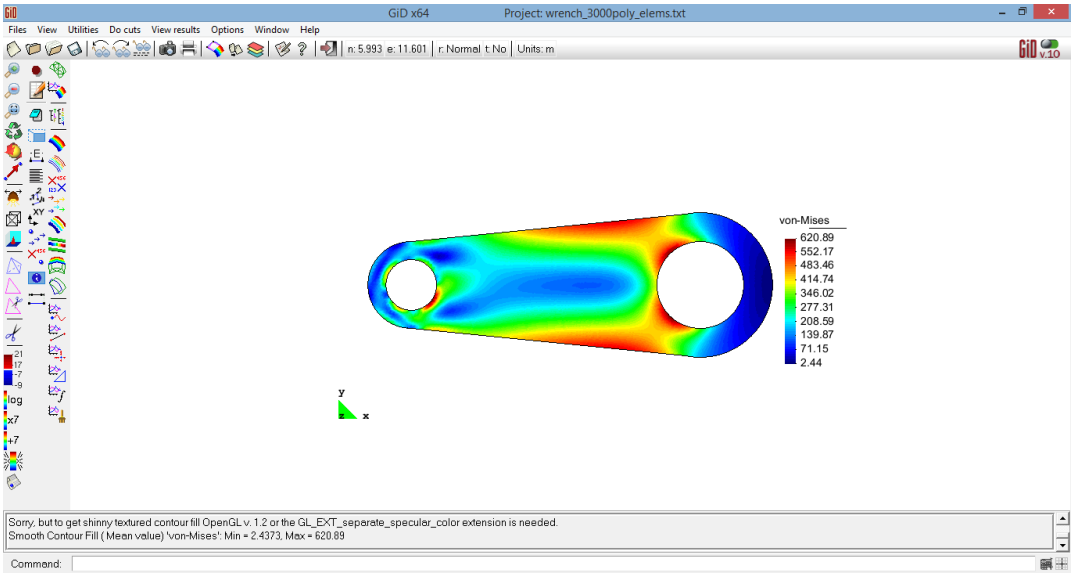

Fig. 6.5: In View results->Smooth Contour Fill, click on von-Mises to plot the smoothed von Mises stress.

Also, GiD can be used to plot deformed shapes as summarized in the following figures.

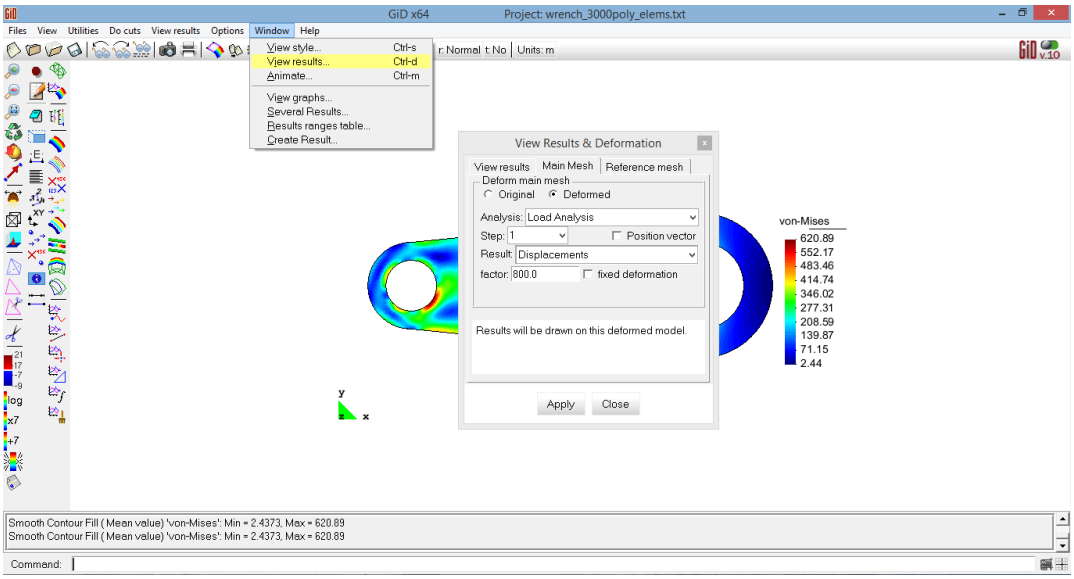

Fig. 6.6: In the postprocess module, click on Window->View results and on the displayed box click on Main Mesh, select Deformed and enter the desired magnification factor in the factor text box.

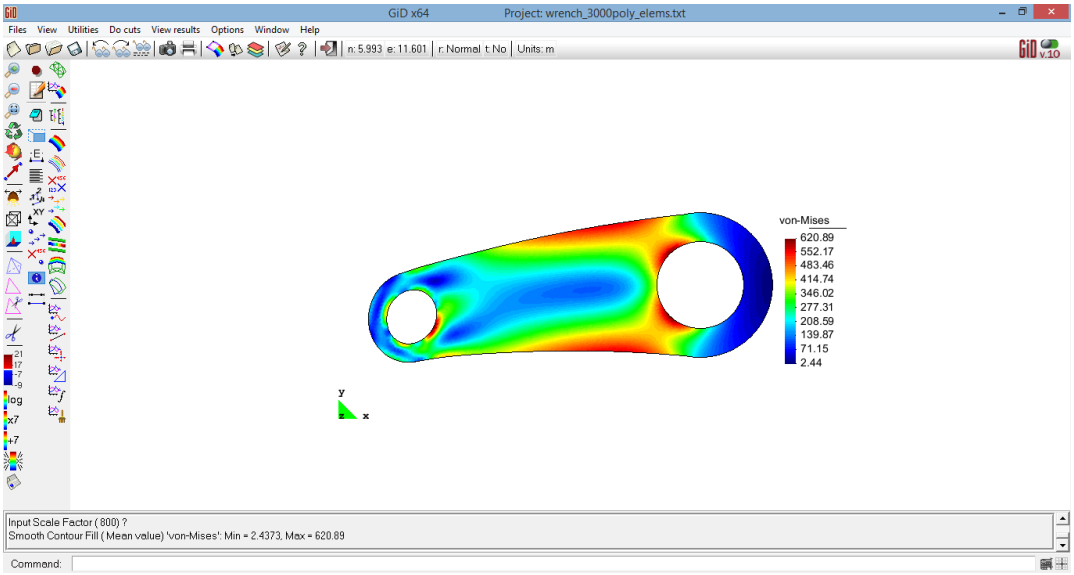

Fig. 6.7: Once the magnification factor is set, click on Apply to display the magnified deformed shape.

7 Setting plot and output options

The plot and output options must be setup by the user. If these are not setup, the program will use the default options. To setup these options go to the file plot_and_output_options.m that is in the folder "config" and activate by setting "yes" or deactivate by setting "no" the available options. Some options include the following (see function plot_and_output_options.m for the complete set of options available):

```
%% GENERAL

create_matlab_contour_plots='yes';
plot_mesh_over_results='yes';
write_solutions_to_text_file='yes';
write_solutions_to_GiD_file='yes';
write_solutions_to_VTK_file='yes';
write_solutions_to_CPP_file='yes';

%% MESH PLOTTING

plot_mesh='yes';           % if yes will plot the mesh
plot_mesh_linewidth = 1.0; % size of the lines in the mesh
plot_mesh_nodes = 'yes';   % if yes will draw a circle at the nodes
plot_mesh_nodesize = 2.0;  % the size of the circle that represents the node
plot_mesh_axis = 'yes';    % if yes will plot the global coordinate system

%% POISSON MODULE

% plotting of main variables to MATLAB/GiD/VTK figures
poisson2d_plot_scalar_field.u='yes';

% plotting of fluxes to MATLAB/GiD figures
poisson2d_plot_flux.qx='no';
poisson2d_plot_flux.qy='no';
poisson2d_plot_flux.qnorm='yes';   % norm of the flux

% plotting of gradients to MATLAB/GiD figures
poisson2d_plot_grad.dx='no';
poisson2d_plot_grad.dy='no';
poisson2d_plot_grad.dnorm='yes';   % norm of the gradient
```

```
%% LINELAST MODULE

% options for plotting deformed domain to MATLAB figures
linelast2d_plot_deformed_domain='yes';
linelast2d_scale_for_plotting_deformed_domain=1; % a number > 1 will scale the
                                                 % deformed domain when plotting to
                                                 % MATLAB figures

% plotting of main variables to MATLAB/GiD/VTK figures
linelast2d_plot_displacement.ux='yes';
linelast2d_plot_displacement.uy='yes';
linelast2d_plot_displacement.unorm='yes';  % norm of the displacement

% plotting of stresses to MATLAB/GiD figures
linelast2d_plot_stress.s11='no';
linelast2d_plot_stress.s12='no';
linelast2d_plot_stress.s22='no';
linelast2d_plot_stress.s33='no';
linelast2d_plot_stress.s1='no';
linelast2d_plot_stress.s2='no';
linelast2d_plot_stress.s3='no';
linelast2d_plot_stress.vm='no';

% plotting of strains to MATLAB/GiD figures
linelast2d_plot_strain.e11='no';
linelast2d_plot_strain.e12='no';
linelast2d_plot_strain.e22='no';
linelast2d_plot_strain.e33='no';
linelast2d_plot_strain.e1='no';
linelast2d_plot_strain.e2='no';
linelast2d_plot_strain.e3='no';
```

8 Running VEMLab in Octave

We are grateful to Dr. Stefan Holst, EMAG Application Manager, Technology Group, CD-adapco/Siemens PLM Software, for his kind advice on making VEMLab to run in Octave.

In Octave, the “computer” function that is at the beginning of each test file, returns a different name than one of those expected by VEMLab when running in MATLAB, i.e., 'PCWIN', 'PCWIN64', 'GLNX86' or 'GLNXA64'. To fix this, at the beginning of each test file simply redefine the variable “opsystem” as follows: opsystem='PCWIN' or opsystem=' GLNX86' if the machine where Octave is installed is a Windows machine or a Linux machine, respectively.

In addition, Octave presents some issues when plotting VEMLab results to MATLAB figures (on small meshes it will do the work, but on larger meshes it will crash). To fix this, switch off all the MATLAB figures by setting the following parameters in the function plot_and_output_options.m that is in the folder “config”:

```
create_matlab_contour_plots='no';
plot_mesh='no';
plot_mesh_over_results='no';
write_solutions_to_text_file='yes';
write_solutions_to_GiD_file='yes';
write_solutions_to_VTK_file='yes';
```

Make sure the last three parameters are set to 'yes' so that one can have access to VEMLab results through text files or can postprocess results in GiD and VTK/Paraview.

9 VEMLab’s website

Check VEMLab’s website for newer versions:

http://camlab.cl/software/vemlab/

--- THE END ---