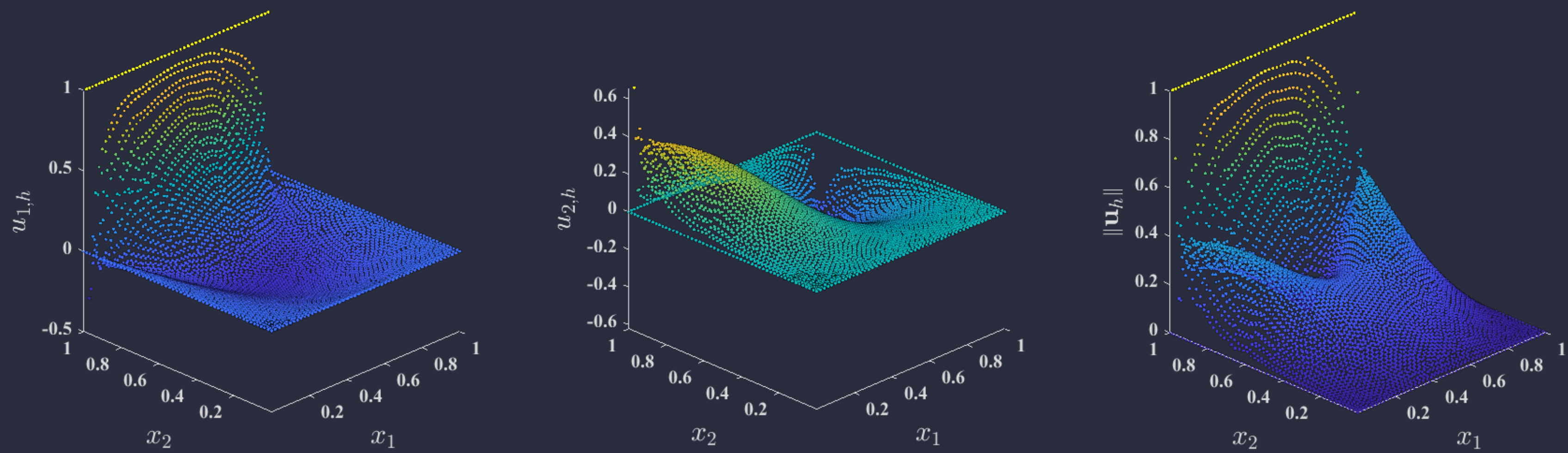
A NODE-BASED UNIFORM STRAIN VIRTUAL ELEMENT METHOD FOR ELASTOPLASTIC SOLIDS



Alejandro Ortiz-Bernardin
Department of Mechanical Engineering
Universidad de Chile
www.camlab.cl/alejandro

Edoardo Artioli
Dipartimento di Ingegneria "Enzo Ferrari"
Università degli Studi di Modena e Reggio Emilia

DIMEC
INGENIERÍA MECÁNICA
UNIVERSIDAD DE CHILE

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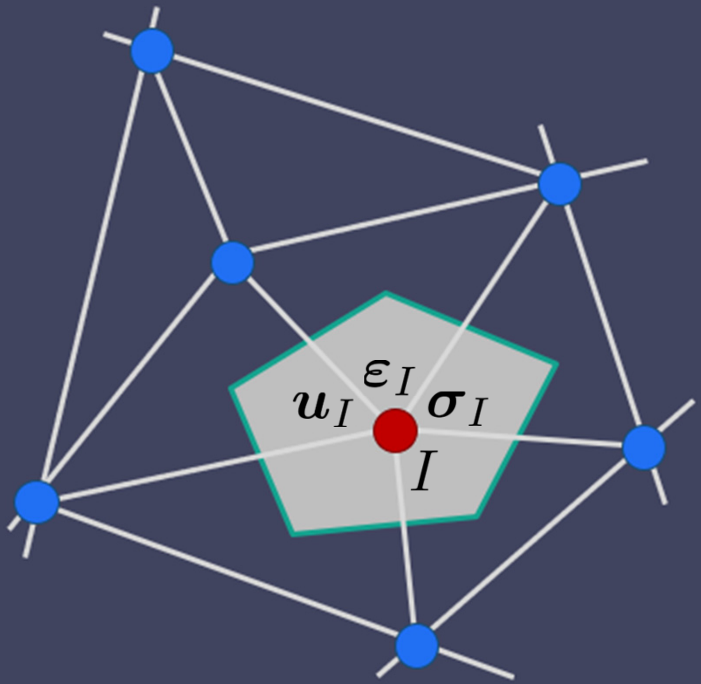
Motivation

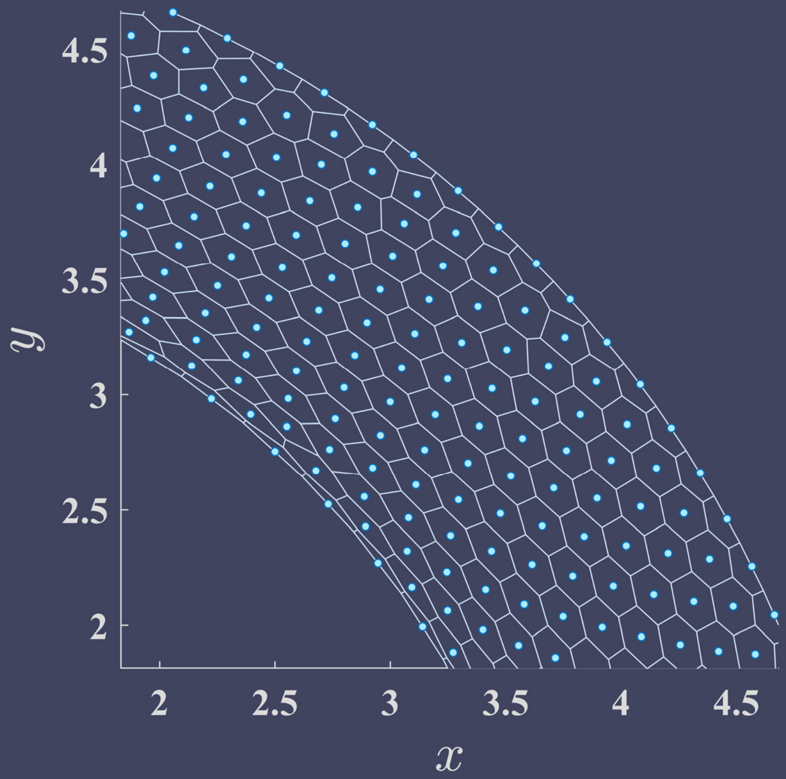


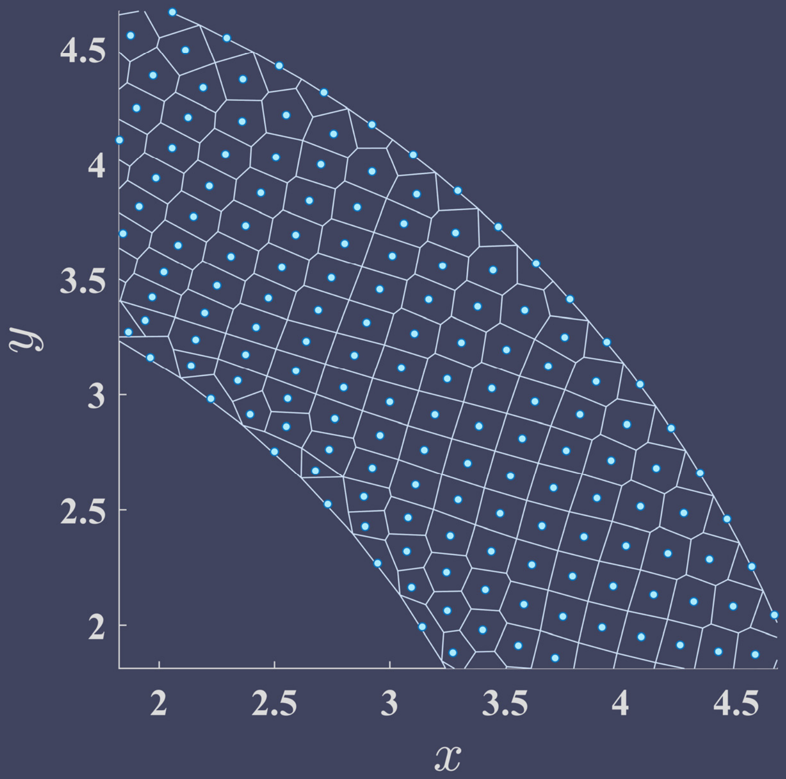
Meshfree nodal integration

What is a nodal integration scheme?

- Numerical integration performed at the nodes
- Displacements, strains and stresses are nodal quantities
- In general, internal and history-dependent variables become nodal quantities

Raison d'être of nodal integration schemes





- Remeshing using the same set of nodes
- Remapping of variables is avoided

Nodal integration + virtual element method: Why?

Meshfree nodal integration

Known for:

- Robustness under mesh distortions
- Locking-free
- Requires stabilization

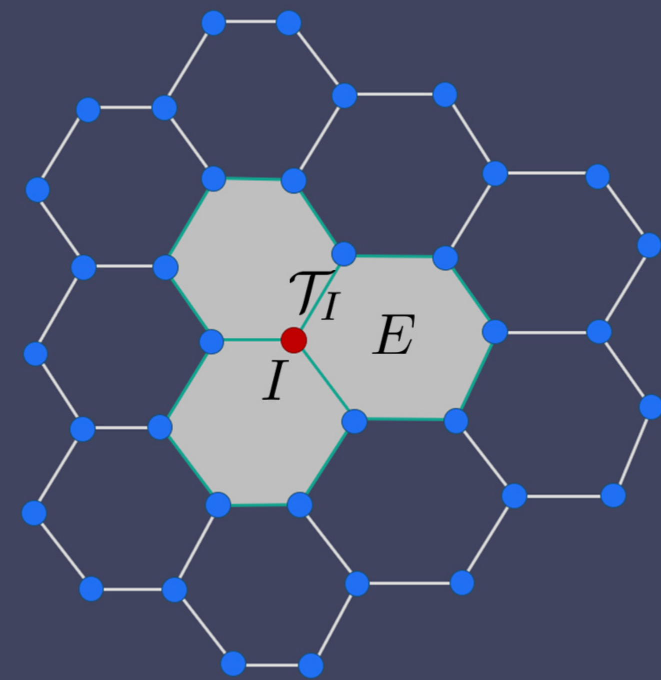
Drawbacks:

- Expensive
- Neighbor search
- Optimization problem

VEM + nodal integration

- Robustness under mesh distortions
- Equipped with a stabilization procedure
- Closer to FEM in terms of computational resources

Node-Based Uniform Strain



Representative nodal area:

$$|I| = \sum_{E \in \mathcal{T}_I} \frac{1}{N_E^V} |E|; \quad N_E^V = \text{\# vertices } E$$

Strain on each virtual element:

$$\widehat{\varepsilon}(\boldsymbol{u}_h) = \frac{1}{2|E|} \int_{\partial E} (\boldsymbol{u}_h \otimes \boldsymbol{n} + \boldsymbol{n} \otimes \boldsymbol{u}_h) \, \mathrm{d}s$$

Node-based uniform strain:

$$\widehat{\varepsilon}_I(\boldsymbol{u}_h) = \frac{1}{|I|} \sum_{E \in \mathcal{T}_I} |E| \frac{1}{N_E^V} \widehat{\varepsilon}(\boldsymbol{u}_h) \implies$$

Nodal averaging operator:

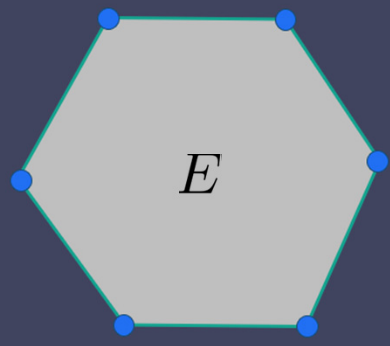
$$\pi_I[F] = F_I = \frac{1}{|I|} \sum_{E \in \mathcal{T}_I} |E| \frac{1}{N_E^V} [F]_E$$

[D2000] Dohrmann et al. Node-based uniform strain elements for three-node triangular and four-node tetrahedral meshes. *International Journal for Numerical Methods in Engineering* 2000; 47(9):1549–1568.

Node-Based Uniform Strain VEM (NVEM)

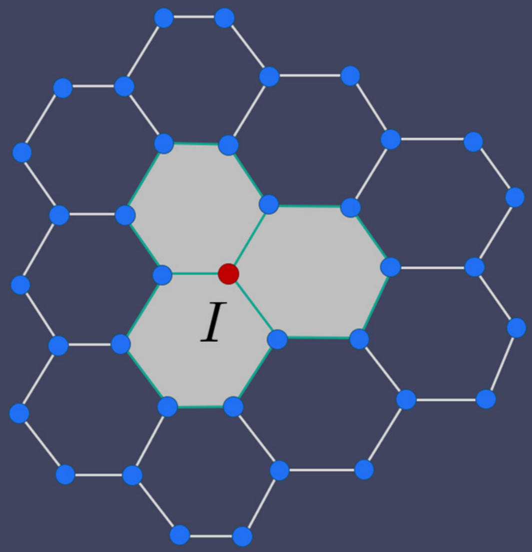
VEM bilinear form at element level:

$$
\begin{aligned}
a_{h,E}(\boldsymbol{u}_h, \boldsymbol{v}_h) &= a_E(\Pi \boldsymbol{u}_h, \Pi \boldsymbol{v}_h) + s_E(\boldsymbol{u}_h - \Pi \boldsymbol{u}_h, \boldsymbol{v}_h - \Pi \boldsymbol{v}_h) \\
&= \sum_E \Big[|E| \, \widehat{\boldsymbol{\varepsilon}}^{\mathsf{T}}(\boldsymbol{v}_h) \, \boldsymbol{D} \, \widehat{\boldsymbol{\varepsilon}}(\boldsymbol{u}_h) \\
&\qquad + (\mathbf{1} - \Pi)^{\mathsf{T}} s_E(\boldsymbol{v}_h, \boldsymbol{u}_h)(\mathbf{1} - \Pi) \Big]
\end{aligned}
$$



NVEM bilinear form at node level:

$$
\begin{aligned}
a_{h,I}(\boldsymbol{u}_h, \boldsymbol{v}_h) &= a_I(\pi_I[\Pi \boldsymbol{u}_h], \pi_I[\Pi \boldsymbol{v}_h]) \\
&\quad + s_I(\pi_I[\boldsymbol{u}_h - \Pi \boldsymbol{u}_h], \pi_I[\boldsymbol{v}_h - \Pi \boldsymbol{v}_h]) \\
&= \sum_I \Big[|I| \, \widehat{\boldsymbol{\varepsilon}}_I^{\mathsf{T}}(\boldsymbol{v}_h) \, \boldsymbol{D} \, \widehat{\boldsymbol{\varepsilon}}_I(\boldsymbol{u}_h) \\
&\qquad + (\mathbf{1} - \Pi)_I^{\mathsf{T}} s_I(\boldsymbol{v}_h, \boldsymbol{u}_h)(\mathbf{1} - \Pi)_I \Big]
\end{aligned}
$$



[OB2023] Ortiz-Bernardin et al. A node-based uniform strain virtual element method for compressible and nearly incompressible elasticity. *International Journal for Numerical Methods in Engineering* 2023; 124(8): 1818–1855.

Stabilization in the NVEM

Stabilization can render the solution too ‘stiff’ in the nearly incompressible limit. To mitigate this, a **modified tangent constitutive matrix** is used in the stability part (like the approach in [P2018]).

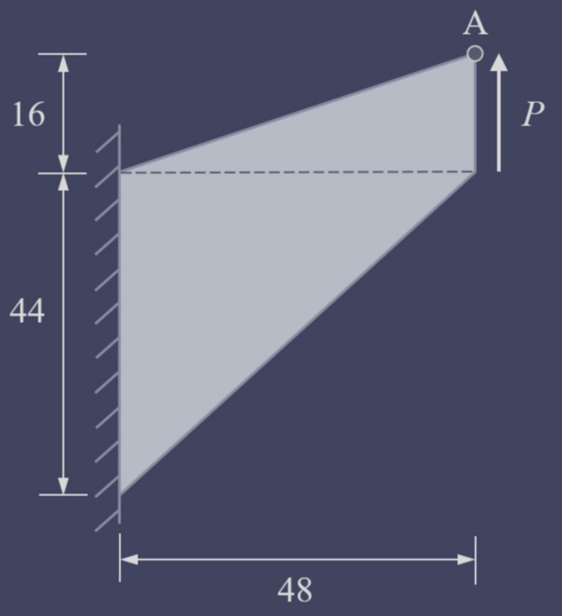
$$(\boldsymbol{s}_I)_{i,i} = \max\left(\left(\boldsymbol{v}_h^{\mathsf{T}}(\boldsymbol{x}_I)\,\boldsymbol{u}_h(\boldsymbol{x}_I)\right)_{i,i},\ \left(|I|\,\widehat{\boldsymbol{\varepsilon}}_I^{\mathsf{T}}(\boldsymbol{v}_h)\,\widetilde{\boldsymbol{D}}_t\,\widehat{\boldsymbol{\varepsilon}}_I(\boldsymbol{u}_h)\right)_{i,i}\right),$$

$$\widetilde{\boldsymbol{D}}_t = \boldsymbol{D}_t\left(\widetilde{E},\widetilde{\nu}\right), \quad \widetilde{E} = \frac{\widetilde{\nu}\left(3\widetilde{\lambda} + 2\widetilde{\mu}\right)}{\widetilde{\lambda} + \widetilde{\mu}}, \quad \widetilde{\nu} = \frac{\widetilde{\lambda}}{2\left(\widetilde{\lambda} + \widetilde{\mu}\right)},$$

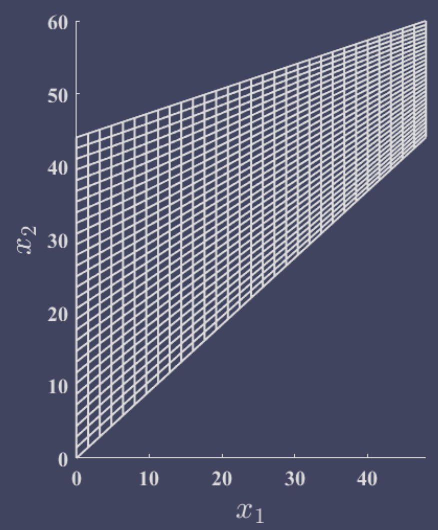
$$\widetilde{\mu} := \min((h_i + h_k)/2,\ 0.05E), \quad \widetilde{\lambda} := \min\left(\lambda, 35\widetilde{\mu}\right)$$

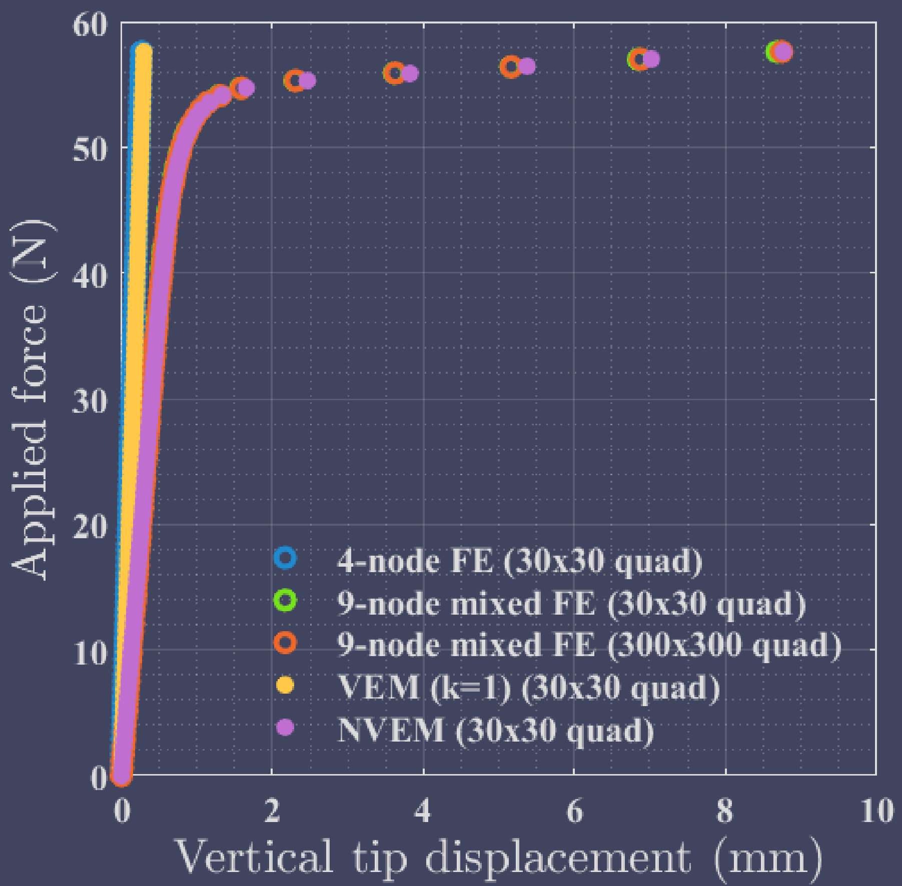
[P2018] Puso et al. Meshfree and finite element nodal integration methods. *International Journal for Numerical Methods in Engineering* 2008; 74(3): 416–446.

Numerical Test: Cook's Membrane



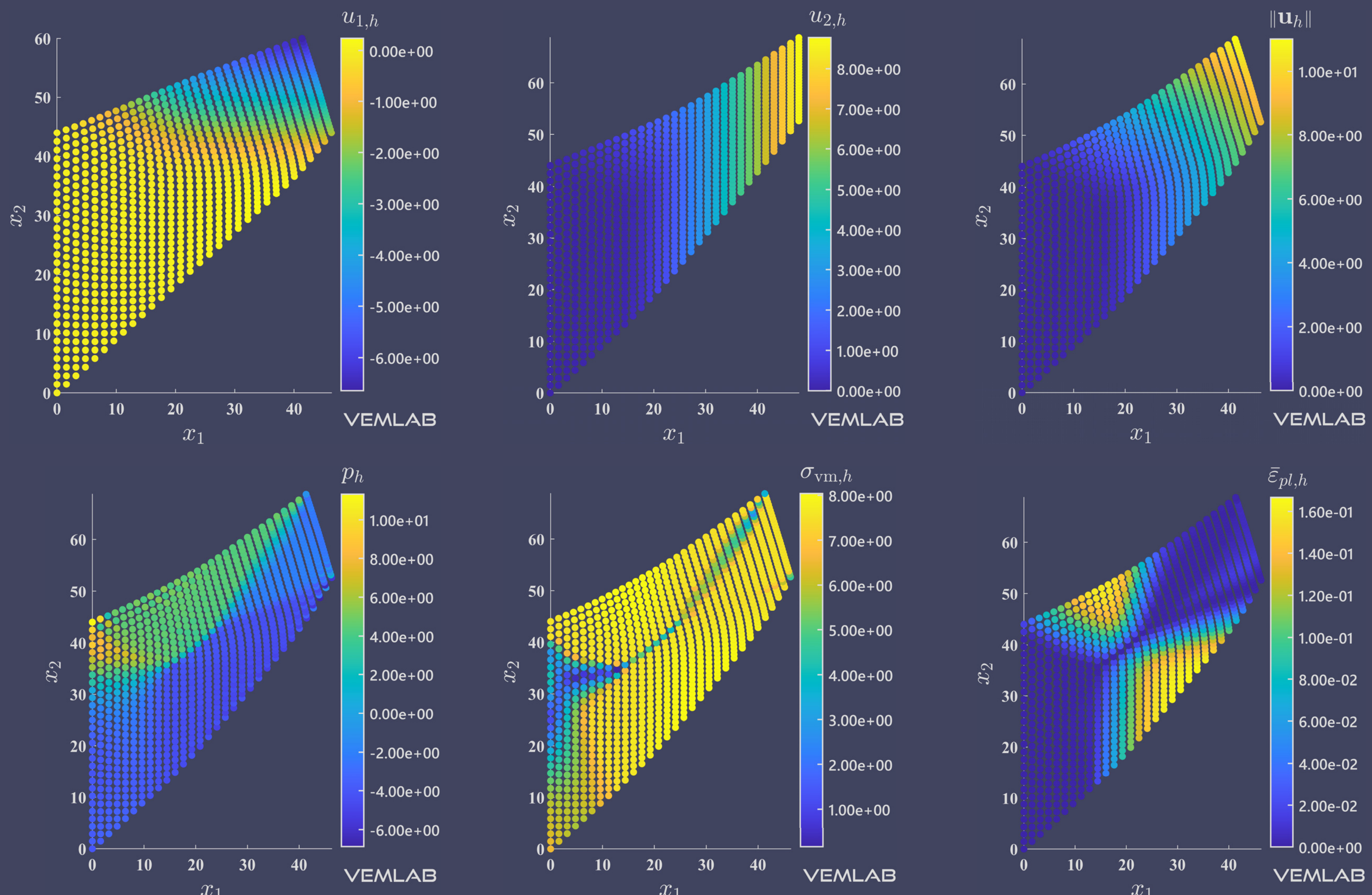
- P = 3.6 N/mm
- $J2$-Plasticity / Plane strain; Isotropic hardening
- E = 1500 Mpa; ν = 0.4999
- H_i = 3.25 Mpa; H_k = 0 Mpa; σ_{y0} = 7.5 MPa





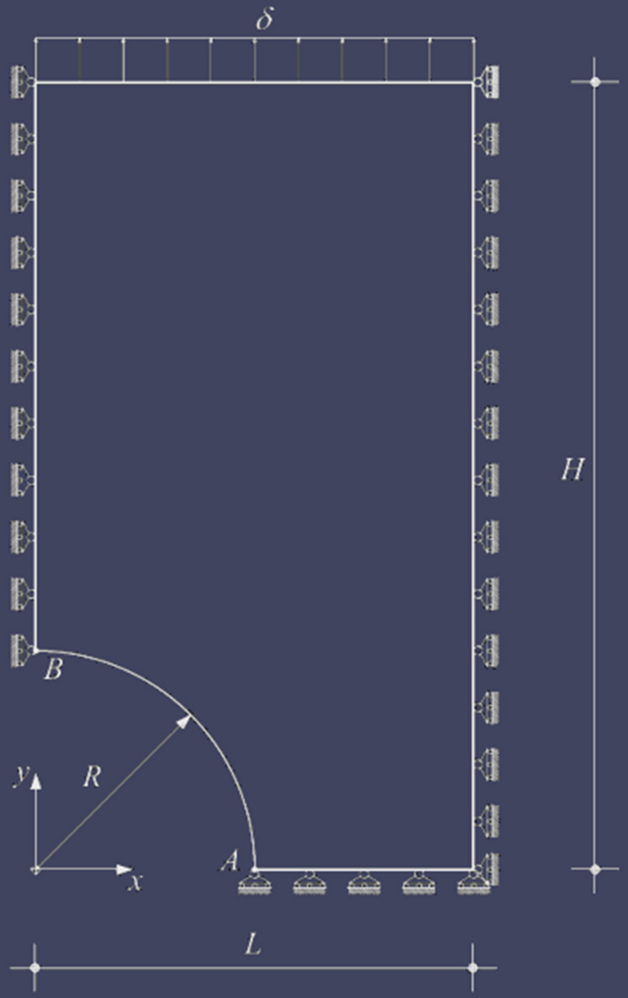
Cook's Membrane (Cont'd)

NVEM Solutions



Numerical Test: Elastoplastic Perforated Plate

(Ref. **[Z2005]**)



Dimensions:

- H = 180 mm
- L = 100 mm
- R = 50 mm

Material:

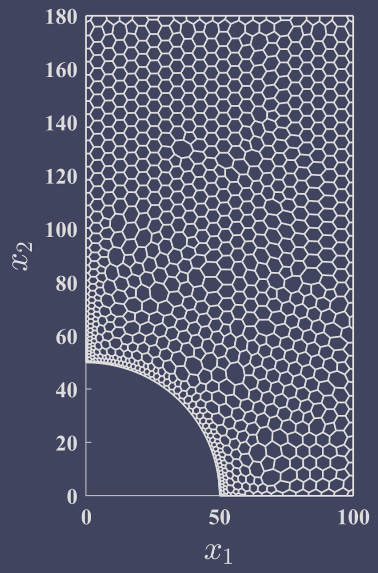
- J2-Plasticity / Plane strain
- Perfect plasticity
- E = 7000 kgf/mm2
- ν = 0.3
- σ_{y0} = 24.3 kgf/mm2

Top displacement:

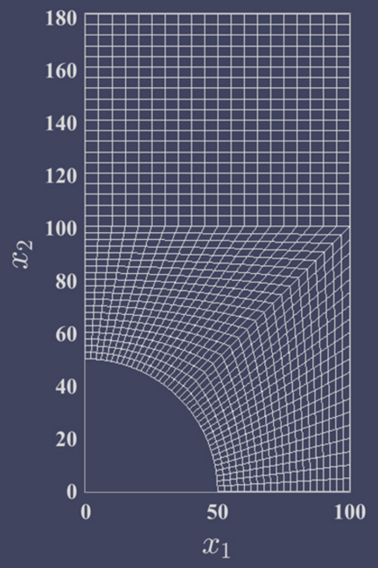
- δ = 2 mm

[Z2005] Zienkiewicz and Taylor. The Finite Element Method for Solid and Structural Mechanics. Sixth Edition, 2005. Elsevier Butterworth-Heinemann.

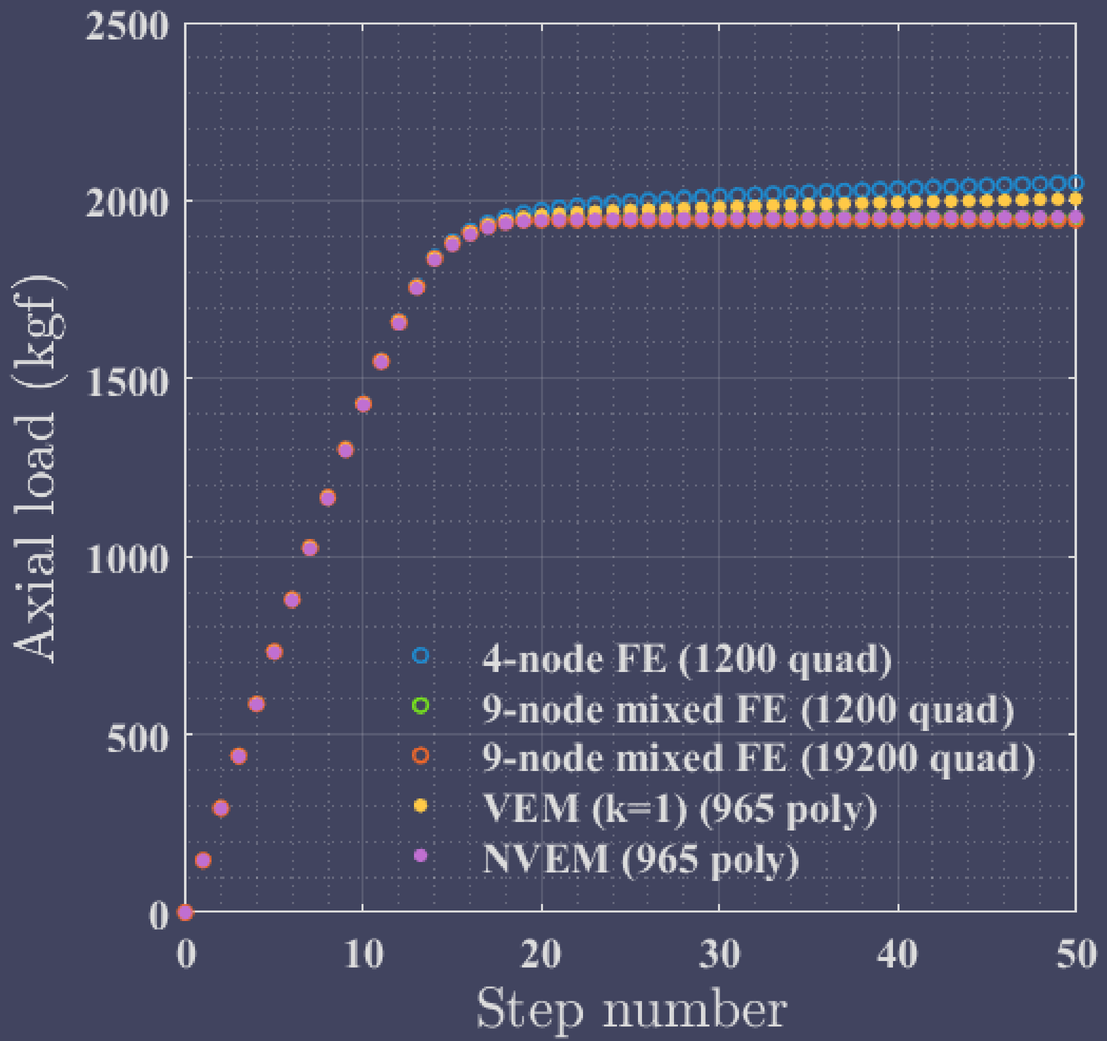
Elastoplastic Perforated Plate (Cont'd)



(965 poly)

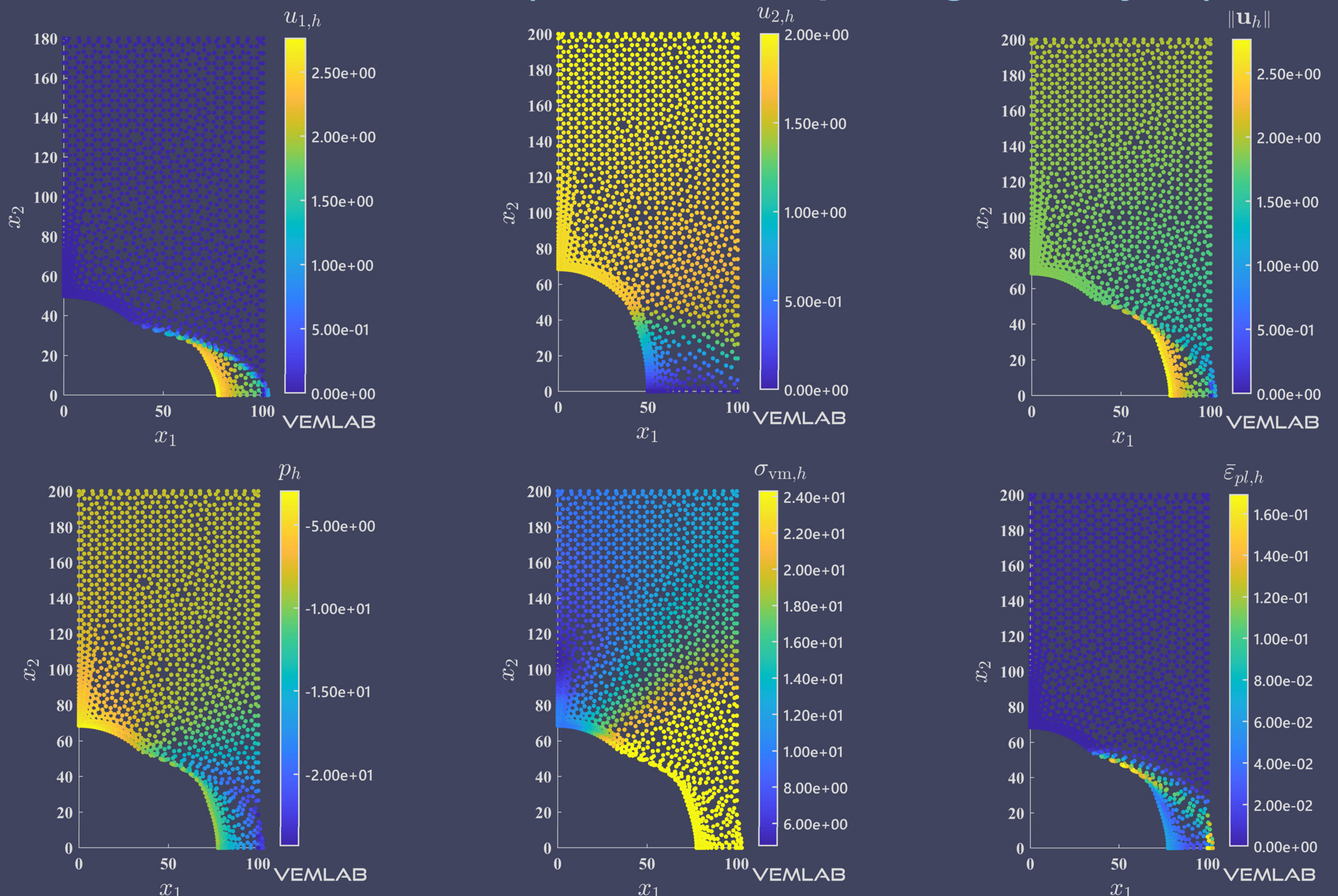


(1200 quad)



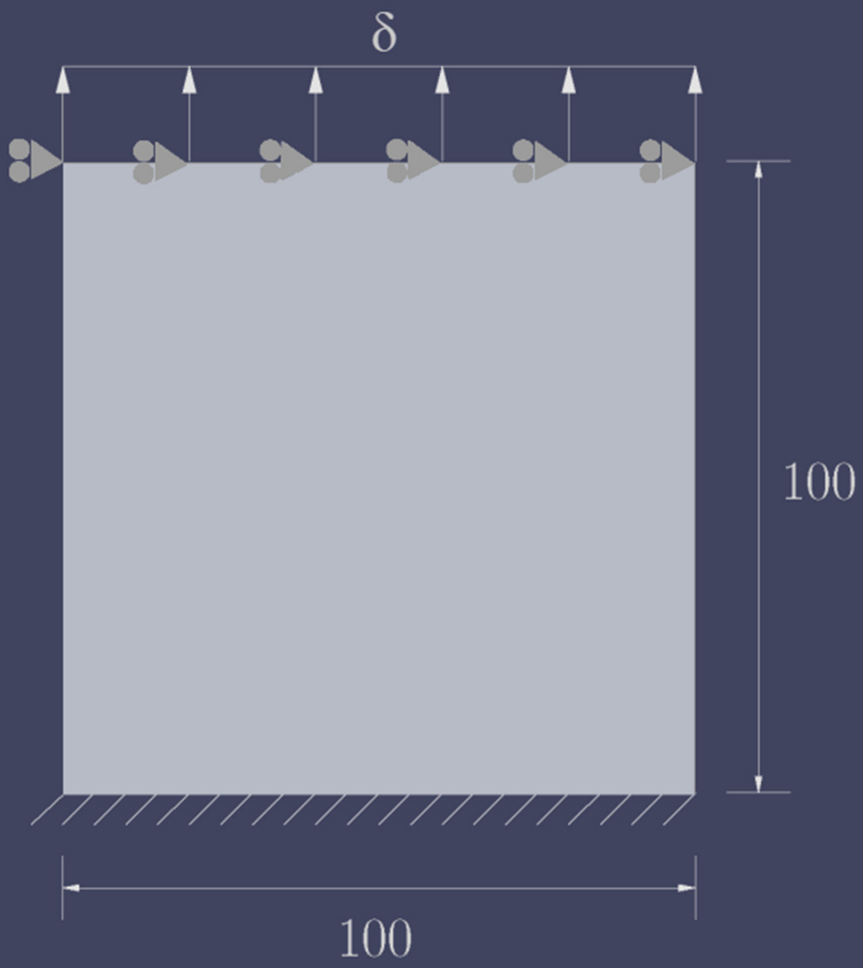
Elastoplastic Perforated Plate (Cont'd)

NVEM Solutions (deformed shape magnified by 10)



Numerical Tests: Tension Benchmark

(Ref. **[C2004]**)



Material:

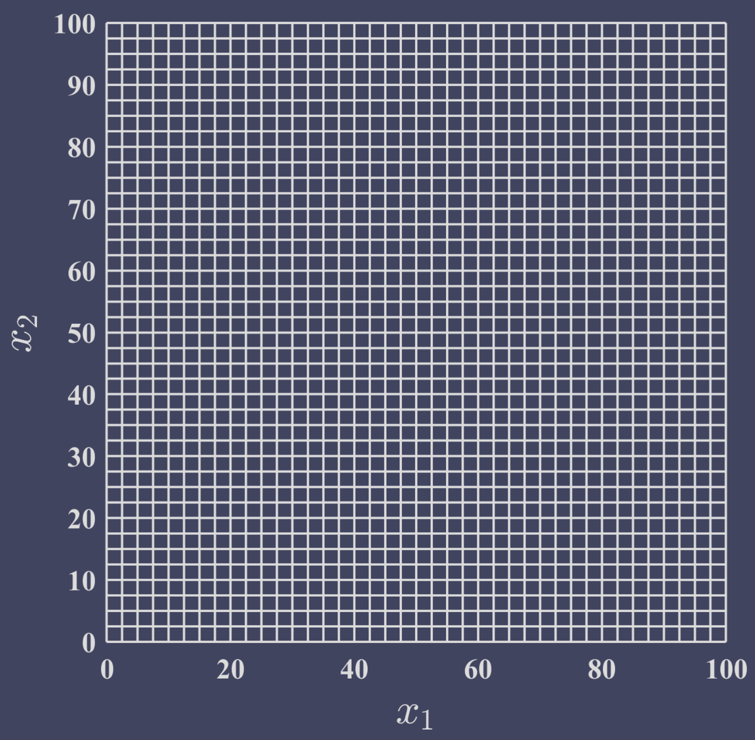
- J2-Plasticity / Plane strain
- Perfect plasticity
- E = 200000 MPa
- ν = 0.4999
- σ_{y0} = 150 MPa

Top displacement:

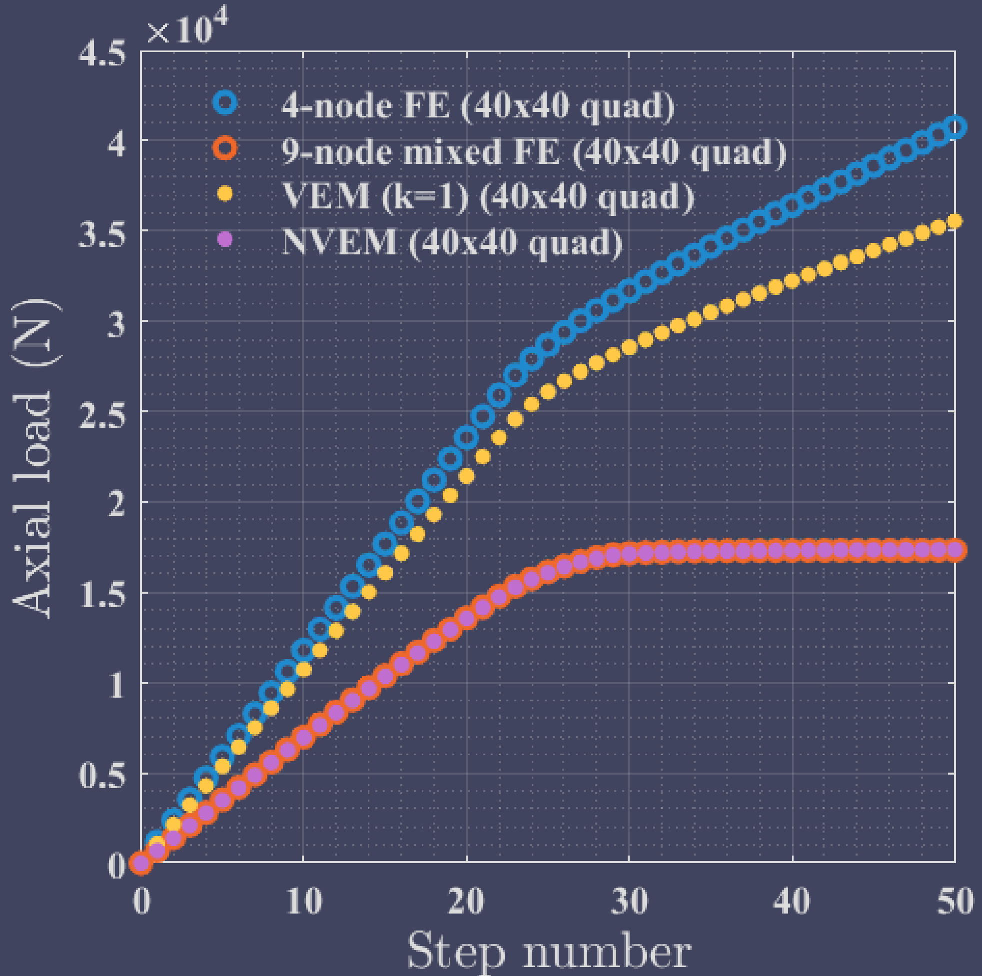
- δ = 0.1 mm

[C2004] Chiumenti et al. A stabilized formulation for incompressible plasticity using linear triangles and tetrahedra. *International Journal of Plasticity* 2004; 20: 1487–1504.

Tension Benchmark (Cont'd)

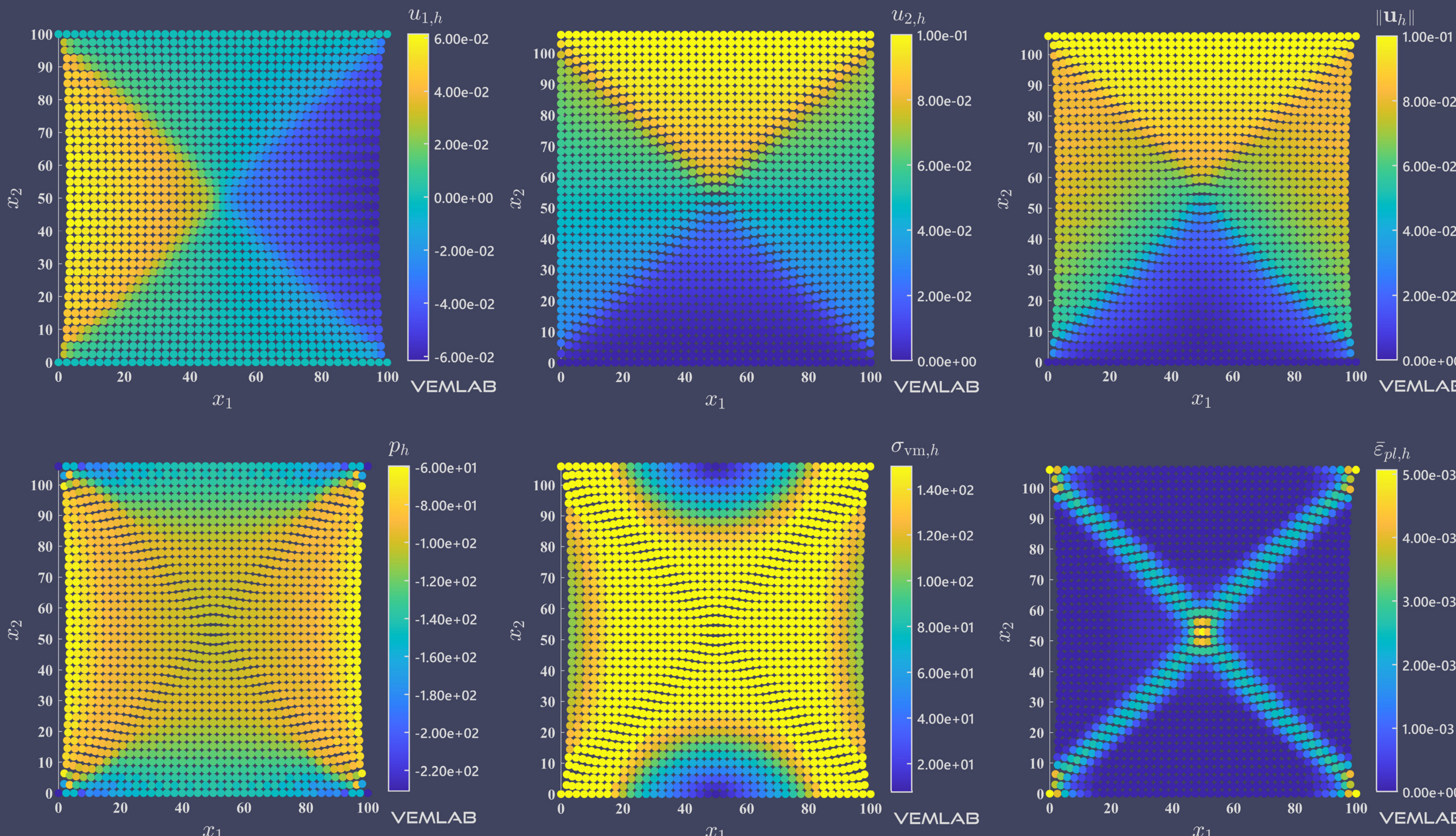


(40x40 quadrilaterals)



Tension Benchmark (Cont'd)

NVEM Solutions (deformed shape magnified by 60)



Summary and Outlook

- Developed NVEM (Nodal Integration – VEM) for elastic and elastoplastic applications.
- So far, NVEM is quite robust for bending and tension dominated problems.
- NVEM is devoid of volumetric locking.
- Outlook: 3D simulations, large deformations with remeshing.

Resources

NVEM for small strain elasticity:

- A node-based uniform strain virtual element method for compressible and nearly incompressible elasticity (https://doi.org/10.1002/nme.7189)

VEM Solver:

- VEMLAB: A MATLAB Library for the Virtual Element Method (https://camlab.cl/software/vemlab/)

Polygonal Mesh Generation:

- POLYLLA: polygonal meshing algorithm based on terminal-edge regions (https://doi.org/10.1007/s00366-022-01643-4)
- Delynoi: an object-oriented C++ library for the generation of polygonal meshes (https://camlab.cl/software/delynoi/)