VIBRATION-BASED DAMAGE ASSESSMENT USING LINEAR APPROXIMATION WITH MAXIMUM ENTROPY

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Abstract. *Supervised learning algorithms have been proposed as a suitable alternative to model updating methods in vibration-based damage assessment, being Artificial Neural Networks the most frequently used. Notwithstanding, the slow learning speed and the large number of parameters that need to be tuned within the training stage have been a major bottleneck in their application. This article presents a new supervised learning algorithm for real-time damage assessment that uses a linear approximation method in conjunction with vibration characteristics measured from the damaged structure. The linear approximation is handled by a statistical inference model based on the maximum-entropy principle. The merits of this new approach are twofold: training is avoided and data is processed in a period of time that is comparable to the one of Neural Networks. The performance of the proposed methodology is validated by considering two experimental structures: an eight-degree-of-freedom (DOF) mass-spring system and an exhaust system of a car.*
Keywords: *Structural damage assessment, supervised learning algorithms, maximum entropy principle, linear approximation*

1. INTRODUCTION

The main problem of vibration-based damage assessment is to ascertain the presence, location and severity of structural damage given the structure's dynamic characteristics. The most successful applications of vibration-based damage assessment are model updating methods based on global optimization algorithms (Meruane and Heylen [2010, 2011], Perera and Torres [2006], Kouchmeshky et al. [2007]). Model updating is an inverse method that identifies the uncertain parameters in a numerical model and is commonly formulated as an inverse optimization problem. In inverse damage detection, the algorithm uses the differences between the models of the structure that are updated before and after the presence of damage to localize and determine the extent of damage. The basic assumption is that the damage can be directly related to a decrease of stiffness in the structure. However, these algorithms are exceedingly slow and the damage assessment process is achieved via a costly and time-consuming inverse process, which presents an obstacle for real-time health monitoring applications.

Supervised learning algorithms are an alternative to inverse modeling. The objective of supervised learning is to estimate the structure's health based on current and past samples. Supervised learning can be divided into two classes: parametric and non-parametric. Parametric approaches assumed a statistical model for the data samples. A popular parametric approach is to model each class density as Gaussian (Markou and Singh [2003]). Nonparametric algorithms do not assume a structure for the data. The most frequently nonparametric algorithms used in damage assessment are Artificial Neural Networks (Gonzalez-Perez and Valdes-Gonzalez [2011], Sahoo and Maity [2007], Arangio and Beck [2012]). A trained neural network can potentially detect, locate and quantify structural damage in a short period of time. Hence, it can be used for real-time damage assessment. Although once the network is already trained it can process data very quickly, the slow learning speed and the large number of parameters that need to be tuned within the training stage have been a major bottleneck in their application. Gupta et al. [2006] presented a new nonparametric method for supervised learning, which generalizes linear approximation by using the maximum-entropy (max-ent) principle (Jaynes [1957]) for statistical inference. A similar approach is adopted by Erkan [2010] for semi-supervised learning problems, where a decision rule is to be learned from labeled and unlabeled data. By using max-ent methods, training is avoided and data is processed in a period of time that is comparable to the one of Neural Networks. In addition, it only requires one parameter to be selected. Hence, max-ent methods become very appealing for real-time health monitoring applications. Gupta [2003] demonstrated the application of the max-ent approach to color management and gas pipeline integrity problems.

In the present paper, we demonstrate the applicability of max-ent methods in structural damage identification. The primary contribution is the development of a real-time damage assessment algorithm that uses a linear approximation method in conjunction with antiresonant frequencies that are identified from transmissibility functions. The linear approximation is handled by a statistical inference model based on the maximum-entropy principle (Jaynes [1957]). The performance of the proposed methodology is validated by considering two experimental structures: an eight-degree-of-freedom (DOF) mass-spring system, and an exhaust system of a car. To demonstrate the potential of max-ent methods over existing ones, results obtained via the max-ent formalism are compared with those of a model updating approach based on parallel genetic algorithms.

2. LINEAR APPROXIMATION WITH MAXIMUM-ENTROPY PRINCIPLE

An observation vector $\mathbf{Y}^j = \left\{Y_1^j, Y_2^j, \ldots, Y_m^j\right\} \in \mathbb{R}^m$ represents the jth damage state of a structure, where m is the number of structural elements. Let the feature vector $\mathbf{X}^j = \left\{X_1^j, X_2^j, \ldots, X_n^j\right\} \in \mathbb{R}^n$ represent a set of characteristics parameters of the structure associated to the damage state $\mathbf{Y}^j$. The variables $\mathbf{X}$ and $\mathbf{Y}$ have joint distribution $P_{X,Y}$. A set of k independent and identically distributed samples be drawn from $P_{X,Y}$, these samples represent the database $(\mathbf{X}^1, \mathbf{Y}^1), (\mathbf{X}^2, \mathbf{Y}^2), ..., (\mathbf{X}^k, \mathbf{Y}^k)$. The central problem in supervised learning is to form an estimate of $P_{Y|X}$, i.e. given a certain feature $\mathbf{X}$ to estimate the corresponding observation $\mathbf{Y}$. Let $\hat{\mathbf{Y}}$ denote the estimated value of $\mathbf{Y}$. Linear approximation takes the N nearest neighbors to a test point $\mathbf{X}$ and uses a linear combination of them to represent $\mathbf{X}$ as

$$\mathbf{X} = \sum_{j=1}^{N} w_j(\mathbf{X})\mathbf{X}^j(\mathbf{X}), \quad \sum_{j=1}^{N} w_j(\mathbf{X}) = 1, \tag{1}$$

where $w_1, w_2, \ldots, w_N$ are weighting functions, and $\mathbf{X}^1(\mathbf{X}), \mathbf{X}^2(\mathbf{X}), \ldots, \mathbf{X}^N(\mathbf{X})$ are the N closest neighbors to a test point $\mathbf{X}$ out of the database set. Typically, equation (1) is tackled via an unconstrained optimization technique of the family of least-squares. However, these methods produce some negative weights, which lacks physical meaning. An alternative that produces positive weights is obtained via the maximum-entropy (max-ent) variational principle (Jaynes [1957]), which can be written as,

$$\max_{w \in \mathbb{R}_+^N} \left[H(w) = -\sum_{i=1}^{N} w_i(\mathbf{X}) \log\left(\frac{w_i(\mathbf{X})}{m_i(\mathbf{X})}\right) \right], \tag{2a}$$

subject to the constraints:

$$\sum_{i=1}^{N} w_i(\mathbf{X})\tilde{\mathbf{X}}^i = 0, \quad \sum_{i=1}^{N} w_i(\mathbf{X}) = 1, \tag{2b}$$

where $m_i(\mathbf{X})$ is a prior distribution that acts as a 'first guess' for $w_i(\mathbf{X})$ and $\tilde{\mathbf{X}}^i = \mathbf{X}^i - \mathbf{X}$ has been introduced as a shifted measure for stability purposes. A typical prior distribution is the smooth Gaussian (Arroyo and Ortiz [2006])

$$m_i(\mathbf{X}) = \exp(-\beta_i \|\tilde{\mathbf{X}}^i\|^2), \tag{3}$$

where $\beta_i = \gamma/h_i^2$; γ is a parameter that controls the radius of the Gaussian prior at $\mathbf{X}^i$, and therefore its associated weight function; and h_i is a characteristic $n-$dimensional Euclidean distance between neighbors that can be distinct for each $\mathbf{X}^i$. In view of the optimization problem posed in (2) for supervised learning, maximizing the entropy chooses the weight solution that commits the least to any one in the database samples (Gupta et al. [2006]). The solution of the max-ent optimization problem is handled by using the procedure of Lagrange multipliers. After $\mathbf{w}$ is obtained, $\hat{\mathbf{Y}}$ is estimated as,

$$\hat{\mathbf{Y}} = \sum_{j=1}^{N} w_j(\mathbf{X})\mathbf{Y}^j(\mathbf{X}), \tag{4}$$

where $\mathbf{Y}^1(\mathbf{X}), \mathbf{Y}^2(\mathbf{X}), \ldots, \mathbf{Y}^N(\mathbf{X})$ are the corresponding observations to the N selected neighbors.

3. CONSTRUCTION OF THE DATABASE

Database samples are generated using a numerical (finite element) model of the structure.

3.1 Feature vector

The jth feature vector $\mathbf{X}^j$ contains the experimental changes in the antiresonant frequencies with respect to the intact case:

$$\mathbf{X}^j = \left\{ \frac{\omega_{1,1}^D - \omega_{1,1}^U}{\omega_{1,1}^U}, \frac{\omega_{2,1}^D - \omega_{2,1}^U}{\omega_{2,1}^U}, \ldots, \frac{\omega_{n_1,1}^D - \omega_{n_1,1}^U}{\omega_{n_1,1}^U}, \ldots, \frac{\omega_{n_r,r}^D - \omega_{n_r,r}^U}{\omega_{n_r,r}^U} \right\}, \tag{5}$$

The superscripts D and U refer to damaged and undamaged, respectively, and $\omega_{i,r}$ is the ith antiresonant frequency of the rth Frequency Response Function (FRF). Experimental antiresonances are identified from transmissibility measurements using the algorithm presented by Meruane [2013].

3.2 Observation vector

The jth observation vector $\mathbf{Y}^j$ contains damage indices represented by elemental stiffness reduction factors, defined as the ratio of the stiffness reduction to the initial stiffness. The stiffness matrix of the damaged structure, $\mathbf{K}_d$, is expressed as a sum of element matrices multiplied by reduction factors:

$$\mathbf{K}_d = \sum_{i=1}^{m} \left(1 - Y_i^j\right) \mathbf{K}_i, \tag{6}$$

where $\mathbf{K}_i$ is the stiffness matrix of the ith element. Thus, $Y_i^j = 0$ indicates that the ith element is undamaged, whereas $0 < Y_i^j < 1$ implies partial damage and $Y_i^j = 1$ complete damage.

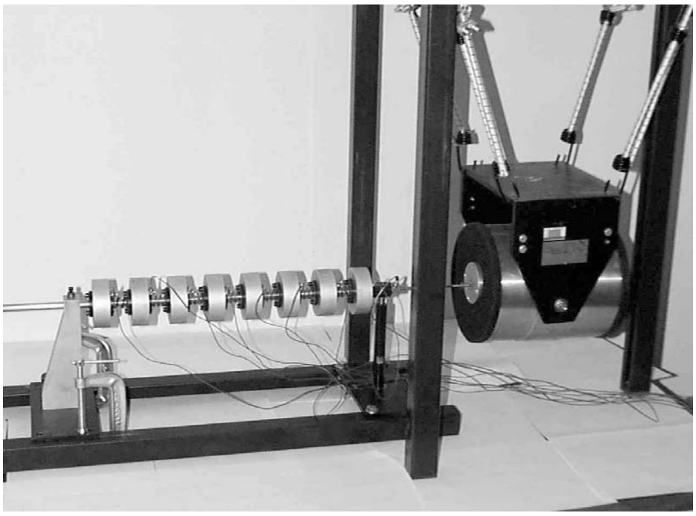
3.3 Distribution of patterns

The distribution of patterns in the database plays a crucial role in the success of the algorithm. The relationship between antiresonant frequencies and different damage levels is not linear. Therefore, the algorithm might not be able to interpolate data. In this study, the patterns were generated by considering single damage scenarios with eight damage levels distributed as 0, 20, 40, 60, 80, 90, 95 and 99.9%.

4. CASE STUDIES

4.1 Eight-DOF spring-mass system

The structure shown in Fig. 1 consists of an eight-DOF spring-mass system. Los Alamos National Laboratory (LANL) designed and constructed this system to study the effectiveness of various vibration-based damage identification techniques (Duffey et al. [2001]). Eight translating masses connected by springs form the system. Each mass is a disc of aluminium with a diameter of 76.2 mm and a thickness of 25.4 mm. The masses slide on a highly polished steel rod and are fastened together with coil springs. The positions of the springs and masses are designated sequentially, with the first ones being closest to the shaker attachment.



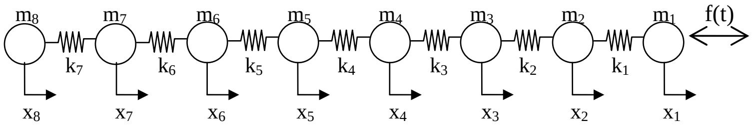


Figure 1. Experimental eight-degrees-of-freedom system.

In the undamaged configuration, all springs are identical and have a linear stiffness coefficient. Damage was simulated by replacing the fifth spring with another spring that has a lower stiffness (55% stiffness reduction). Acceleration was measured horizontally at each mass yielding eight measured DOFs. The structure was excited randomly by an electrodynamic shaker. Twenty-eight antiresonant frequencies were identified from the transmissibility measurements.

The numerical model was built in `Matlab`® with springs and concentrated masses. The database samples were created using the seven springs as possible locations for damage, resulting in 56 patterns.

During the set-up of the damage identification algorithm, the only parameter that needs to be selected in the max-ent computation is γ for the Gaussian prior. This parameter controls the radius of the weight functions associated with those components of the feature vector $\mathbf{X}^j$ that contribute to the approximation at the feature test vector $\mathbf{X}$. Therefore, it determines the number of neighbors to the test point. Fig. 2 shows the results for the experimental damage case using $\gamma = 1600$. The results are compared with those obtained using a model updating approach based on Parallel Genetic Algorithms (PGA) Meruane [2013]. Both algorithms were able to correctly identify the experimental damage represented by a 55% stiffness reduction in element 5. In terms of time, the PGA approach required 206 seconds to yield a solution, whereas the linear approximation max-ent algorithm required only 0.065 seconds.

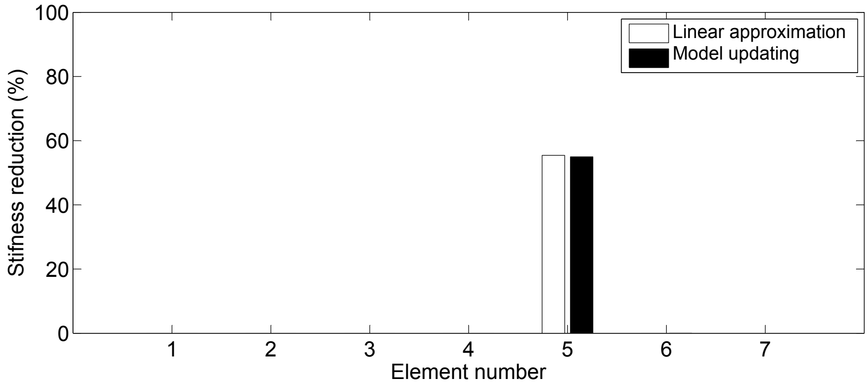


Figure 2. Identification of experimental damage in the eight DOF mass-spring system using two algorithms: linear approximation with maximum entropy and model updating with parallel genetic algorithms.

4.2 Car Exhaust System

The structure consists of a car exhaust system as shown in Fig. 3. The dimensions are: length: 2.3 m, width: 0.45 m. The exhaust pipe has a diameter of 38 mm. The structure is suspended by soft springs and is excited randomly by an electrodynamic shaker. The response is captured by 16 accelerometers. The test is performed in a frequency range of $0 - 512$ Hz with a frequency resolution of 0.25 Hz.

The numerical model was built in `Matlab`® with $2D$ beam elements and concentrated inertias for the masses. The model has 47 beam and 5 inertia elements, with 144 degrees of freedom. Elements 18 to 47 are considered possible locations of damage, giving 30 damage locations and 240 patterns.

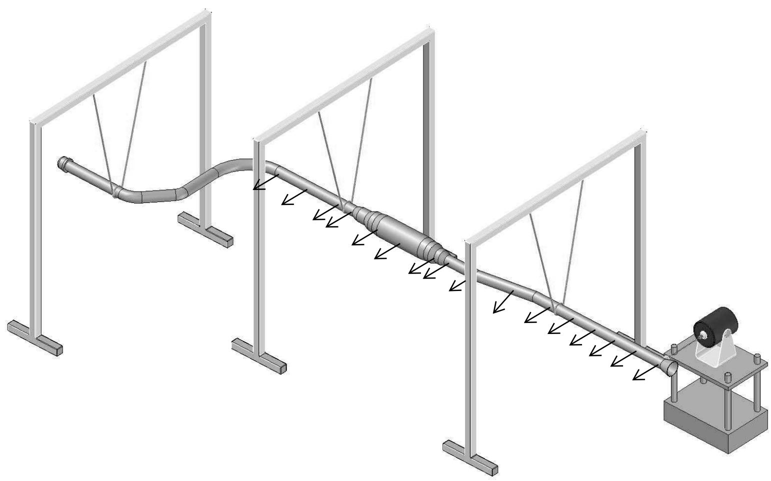


Figure 3. Experimental set-up of the car exhaust system.

A single fatigue crack with three increasing levels of damage is introduced to the structure. Fig. 4-a) shows the crack, which is located in element 31, close to the welded connection between elements 30 and 31 and covers around 60% of the pipe perimeter. The fatigue test is done again twice to grow the crack. Fig. 4-b) shows the second damage level; here the structure has already failed due to unstable crack propagation. The open crack covers around 70% of the perimeter. The last damage level is shown in Fig. 4-c). The crack covers around 85% of the perimeter.

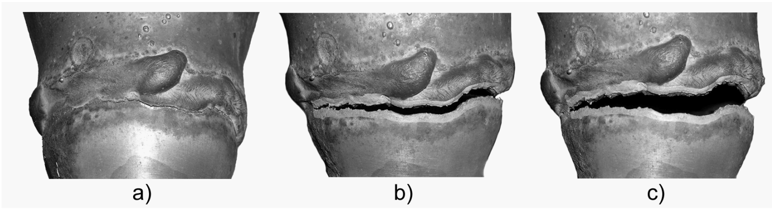


Figure 4. Three levels of damage introduced to the car exhaust system.

The algorithm automatically selects an appropriate value for γ in the Gaussian prior. It starts with a value of $\gamma = 500$ and increases its value by increments of 200 until the number of neighbors patterns contributing to the solution is equal or less than 4. Fig. 5 shows the results of the experimental damage identified by both approaches: the linear approximation with maximum entropy and a model updating based on parallel genetic algorithms. An arrow indicates the actual damage location. In the three cases, the damage is correctly identified by both approaches. Though, in the first case the damage detected by the linear approximation approach is closer to the actual location.

In terms of time, the PGA approach requires approximately 1800 seconds to assess the experimental damage in each case, whereas the linear approximation approach requires only 0.7 seconds.

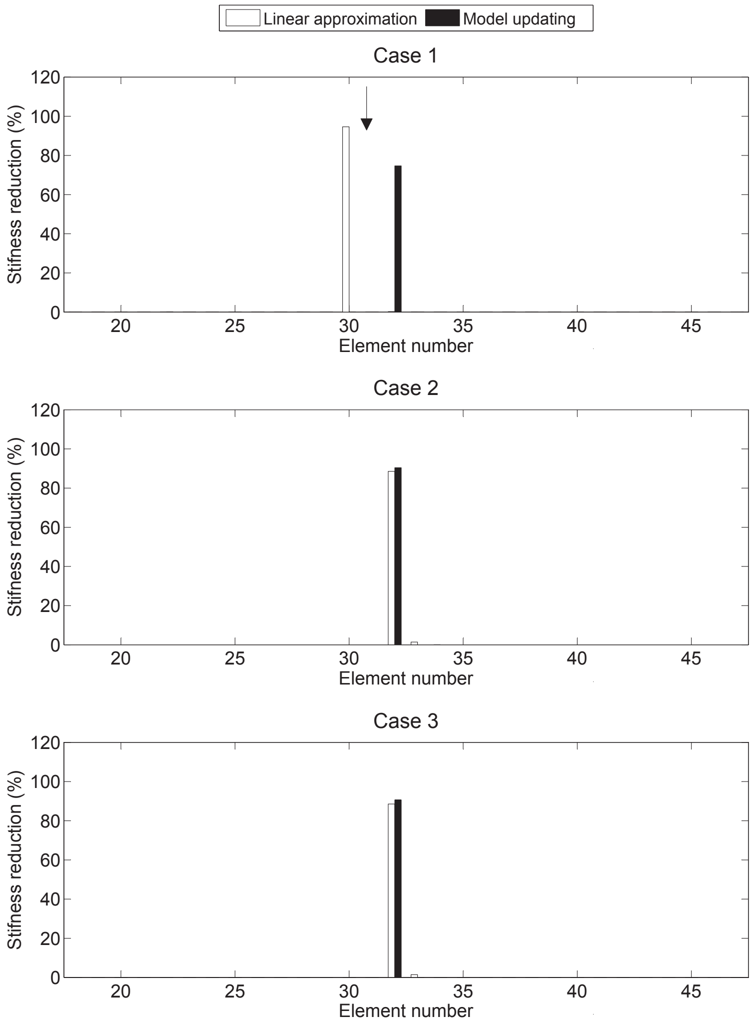


Figure 5. Identification of experimental damage in the car exhaust system using two algorithms: linear approximation with maximum entropy and model updating with parallel genetic algorithms.

5. CONCLUSIONS

This article presented a new supervised learning algorithm for real-time damage assessment that uses a linear approximation method in conjunction with antiresonant frequencies that are identified from transmissibility functions. The linear approximation is handled by a statistical inference model based on the maximum-entropy principle. The performance of the proposed methodology was validated by considering two experimental structures: an eight-DOF mass-spring system and an exhaust system of a car.

In the two structures, the linear approximation using the max-ent technique was successful in assessing the experimental damage. The detected damage closely corresponds to the experimental damage in all cases, obtaining results very similar to those of a model updating approach based on parallel genetic algorithms. The linear approximation approach was able to assess the experimental damage in less than one second in all cases, whereas the model updating approach required between 3 to 30 minutes. Hence, the proposed algorithm provides the possibility of continuously monitoring the state of a structure.

The linear approximation method, as presented, is able to accurately assess single damage scenarios. Further research is needed to adapt this algorithm to cases with multiple damage scenarios and to test its performance with more complex structures.

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